

AMERICAN JOURNAL of PHYSICS

Devoted to the Instructional and Cultural Aspects of Physical Science

VOLUME 22

NUMBER 3

MARCH 1954



Published nine times per year for the
AMERICAN ASSOCIATION OF PHYSICS TEACHERS
by the
AMERICAN INSTITUTE OF PHYSICS
Incorporated

PRINCE & LEMON STS., LANCASTER, PA., or 57 EAST 55 STREET, NEW YORK 22, N. Y.

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Manuscripts for publication should be submitted to Dr. Thomas H. Osgood, *American Journal of Physics*, School of Graduate Studies, Michigan State College, East Lansing, Michigan. The authors' institutions are requested to pay an optional publication charge of \$4 per page which, when honored, entitles them to 100 free reprints. Instructions will be sent with galley proofs.

Proof and correspondence concerning papers in the process of being printed should be addressed to the American Institute of Physics, 57 East 55 Street, New York 22, New York.

Subscription Price

<i>United States and Canada</i>	\$6.00
<i>Elsewhere</i>	7.00

Back Numbers

Complete set: Vol. 1, 1933-Vol. 21, 1953—\$147.00

Yearly back number rate: \$8.00

Single copies: \$2.00 each 1933-1936; \$1.50 each thereafter.

Cumulative index (Vols. 1-20, 1933-1952): \$3.00

Subscriptions, renewals, and orders for back numbers should be addressed to the American Institute of Physics, 57 East 55 Street, New York 22, New York. Members and junior members of the American Association of Physics Teachers receive the *American Journal of Physics*.

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The contents of the *American Journal of Physics* can be found indexed in the *Education Index*.

AMERICAN JOURNAL of PHYSICS

A Journal Devoted to the Instructional and Cultural Aspects of Physical Science

VOLUME 22, NUMBER 3

MARCH, 1954

Reproductions of Prints, Drawings, and Paintings of Interest in the History of Physics

59. Title Page from Alhazen's *Opticae Thesaurus*

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(Received August 17, 1953)

Mediaeval optics was dominated by the *Kitāb-al-manāzur* of Ibn al-Haitham. The Latin translation published at Basel in 1572 and known from its Latin title as the *Opticae Thesaurus* of Alhazen has an interesting woodcut on its title page which is here reproduced.

THE Egyptian ALHAZEN—to use the more familiar Latinized name of IBN AL-HAITHAM (*ca* 965–1039)—was, according to SARTON, “the greatest Muslim physicist and one of the greatest students of optics of all times.” He was the first great discoverer in optics after the time of PTOLEMY (second century A.D.) and EUCLID (*ca* 300 B.C.), and his great book, the *Kitāb-al-manāzur*, was the chief source of most that was known about optics from the tenth until the beginning of the seventeenth century in England, France, Flanders, Germany, Poland, Italy, and even Persia. It was translated into Latin by an unknown writer of the late twelfth or early thirteenth century and is generally known by its abbreviated Latin title, *Opticae Thesaurus Alhazeni Arabis*. The Latin translation after careful editing was published by FREDERIC RISNER at Basel in 1572 together with the *Opticae Libri decem* of the Polish physicist, WITELO, (born *ca* 1230) under the title *Opticae Thesaurus Alhazeni Arabis libri septem, nunc primum editi. Eiusdem liber de crespuculis et*

Triplcis visus, directi, reflexi & refracti, de quo optica disputat, argumenta.



FIG. 1. Title page from 1572 edition of *Opticae Thesaurus*.

nubium ascensionibus. Idem Vitellonis Thuringopoloni libri X.

That this work was indeed the connecting link between Greco-Arabic optics and the West is happily suggested by the full-page woodcut on its title-page which is here reproduced. (See Fig. 1.) In addition to summarizing in a spectacular manner much of optical knowledge of the Greeks and Arabs—the theory of the rainbow, the use of burning mirrors, reflection, refraction—it portrays a bridge connecting the East and West across which the wisdom of the East, shown

allegorically by means of elephants,¹ passes to the West. It is possible also that the body of water shown is intended to be the Mediterranean and that the mountains in the background are the Atlas range.

Summaries of ALHAZEN'S contributions to optics will be found in several of the histories of physics, that in E. GERLAND'S *Geschichte der Physik* (Munich and Berlin, 1913), pp. 161-170, being particularly recommended.

¹ According to Richard Payne Knight, *The Symbolical Language of Ancient Art and Mythology* (New York, 1892), p. 136, "the elephant has from time immemorial been the symbol of divine wisdom among the Hindus."

Space Charge between Parallel Planes

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(Received July 17, 1953)

The problem of the potential distribution between two parallel planes at the same potential as a function of the current density passing between the planes is analyzed by successive approximation resulting in power series. The method involves a consideration of the influence of space charge on transit time, and the leading terms of the Maclaurin's series expression for the transit time in powers of the current density are evaluated.

FOR illustrating the fundamental laws of electrostatics the teacher of physics often seeks particularly simple charge distributions which may be analyzed without unnecessary mathematical complications. A real difficulty in this connection is that the distributions of charge used to illustrate the theory may seem artificial and far removed from situations arising in practice. The space charge existing in the grid-anode region of a vacuum tube is of considerable interest because of its extensive and important uses. Although certain idealizations are introduced to simplify the analysis, they do not vitiate the results nor do they make the problem introduced in this connection appear artificial.

Many theoretical treatments of the space charge encountered in thermionic vacuum tubes have been published. For parallel planes and complete space charge Langmuir¹ first established the $\frac{3}{2}$ power law for the dependence of current on potential difference. Langmuir and

Blodgett,^{2,3} using power series, provided a solution for the same problem for coaxial cylinders, and for concentric spheres. The practical importance of coaxial cylinders has led to extensive theoretical work with them. Recently Page and Adams⁴ have provided a new and in many ways a more convenient solution of the problem of space-charge limited current between coaxial cylinders. They have also provided a complete solution for the potential distribution between coaxial cylinders with any degree of space charge and also any initial velocity. Copeland and Eggenberger⁵ have analyzed the same problems using the behavior of the transit time as the method of approach.

The effects of space charge in the grid-anode region have been analyzed by Harris,⁶ by Plato,

¹ I. Langmuir and K. B. Blodgett, Phys. Rev. 22, 347 (1923).

² I. Langmuir and K. B. Blodgett, Phys. Rev. 24, 49 (1924).

³ L. Page and N. I. Adams, Phys. Rev. 76, 381 (1949).

⁴ P. L. Copeland and D. N. Eggenberger, J. Appl. Phys. 23, 280 (1952).

⁵ J. H. O. Harris, Wireless Engr. 13, 190 (1936).

¹ I. Langmuir, Phys. Rev. 2, 450 (1913).

Kleen, and Rothe,⁷ by Salzberg and Haeff,⁸ and by Fay, Samuel, and Shockley.⁹ As a result of this work solutions of the basic problems are known. The mathematical formulations, however, may obscure some of the physics that is of interest. Also, it is fair to state that solutions giving the potential as a function of distance where the parameter determining the shape of the curve is the ratio of the screen potential to the value of the potential at the minimum are not very convenient for use. One ordinarily does not know the value of the potential at the potential minimum. Obtaining the proper solution to the problem ordinarily involves a certain amount of trial and error.

What one ordinarily knows is the potentials of both electrodes bounding the space and the value of the transmitted current. It would be desirable to have a method for determining the potential distribution directly from these quantities. The development of such a method is the purpose of this paper. However, we will not give here a general solution but for the purpose of illustrating the method we will restrict our consideration to the most simple case. The grid will be considered a perfectly transparent equipotential plane at the same potential as the anode. We shall also consider only the case in which charge moves in a single direction through the region being investigated.

The work of Copeland and Eggenberger⁶ provides the methods which we shall apply to this problem. If the uniform current density is represented by J , the element of charge dQ contained in unit cross section of the beam over an element of the path for which dt represents the transit time is given by

$$dQ = Jdt. \quad (1)$$

(We use cgs electrostatic units throughout this paper.) In the paper cited Copeland and Eggenberger treat the problem of space charge between an ideal cathode and an anode by considering that for small current the potential distribution between the electrodes may be approximated by the space-charge-free distribu-

tion for the purpose of computing dt for various elements of the path. Thus

$$dt = ds(m/2eV_0)^{\frac{1}{2}},$$

where ds is an element of path along which the electron moves, dt is the element of time corresponding to the passage of an electron along ds , e/m is the ratio of the charge to the mass of the electron, and V_0 is the potential distribution unmodified by space charge. For low currents a very good approximation to the charge is

$$dQ = Jds(m/2eV_0)^{\frac{1}{2}},$$

and since we are considering unit cross-sectional area, ds gives the volume occupied by this charge, and ρ_0 the space-charge density is given by

$$\rho_0 = J(m/2eV_0)^{\frac{1}{2}}. \quad (2)$$

The application of Poisson's equation will give us the potential distribution corrected to take account of this charge density.

Since in the approximation represented by Eq. (2) ρ_0 is a constant, the solution of this problem illustrates the behavior of potential in the case of this very simple distribution of charge. We consider an ideal grid in a plane and spaced at a distance D from the grid an anode in a parallel plane. The current injected at the grid is considered to be of uniform density J over the entire plane. We let x be a Cartesian coordinate perpendicular to the electrodes and take Poisson's equation in the form

$$d^2V/dx^2 = -4\pi(-\rho_0) = 4\pi\rho_0, \quad (3)$$

where V is the electrostatic potential and ρ_0 is the magnitude of the charge per unit of volume.

If in Eq. (3) we use the constant ρ_0 , integrations may be performed without difficulty

$$dV/dx = 4\pi\rho_0x + C_1.$$

Because of the symmetry existing in the problem, it is clear that dV/dx will be zero at the midpoint between the planes, and if the coordinate x is measured from this point, the constant of integration is zero. The result of the second integration therefore is

$$V = 2\pi\rho_0x^2 + C_2.$$

⁷ Plato, Kleen, and Rothe, Z. Physik 101, 509 (1936).

⁸ B. Salzberg and A. V. Haeff, RCA Rev. 2, 336 (1938).

⁹ Fay, Samuel, and Shockley, Bell System Tech. J. 17, 49 (1938).

When $x = D/2$, $V = V_0$. Hence

$$C_2 = V_0 - 2\pi\rho_0(D/2)^2 = V_0 - \frac{1}{2}\pi\rho_0 D^2.$$

Thus

$$V = V_0 - \frac{1}{2}\pi\rho_0 D^2 + 2\pi\rho_0 x^2.$$

By the use of a moving charge cloud we are able in practice to secure a very nearly uniform charge distribution. The tendency of the charge to spread due to mutual interactions between electrons has been minimized because, for an electron moving through the space, the net force due to all electrons acts first in one direction and then for an equal time acts in the opposite direction. In this problem since the force due to the space-charge cloud first acts to retard the electron and then to accelerate it, the effect is to produce some concentration of the charge about the midplane. The approximate amount of this variation is found by replacing the V_0 in Eq. (2) by V

$$\rho = J(m/2eV_0)^{\frac{1}{2}} / \left(1 - \frac{\pi\rho_0 D^2}{2V_0} + 2\frac{\pi\rho_0 x^2}{V_0} \right)^{\frac{1}{2}},$$

and the relative variation in the charge is

$$\frac{\Delta\rho}{\rho} = \frac{\pi\rho_0 D^2}{4V_0}.$$

If the current is very much less than the maximum current which space charge will permit to pass through the space at this energy, the relative variation of the space-charge density is quite small and we have a problem illustrating the potential distribution due to a practically uniform distribution of charge between plane parallel electrodes.

As a convenient means of specifying the current, we define a parameter δ by the following

$$\delta \equiv J/J_s. \quad (4)$$

In this relation J_s is the space-charge limited current given by

$$\begin{aligned} J_s &= \frac{(2e/m)^{\frac{1}{2}}}{9\pi} \frac{V_0^{\frac{1}{2}}}{(D/2)^2} \\ &= \frac{4(2e/m)^{\frac{1}{2}}}{9\pi} \frac{V_0^{\frac{1}{2}}}{D^2}. \end{aligned} \quad (5)$$

Since to terms of the order ρ_0^2 , the charge density is given by

$$\rho = \rho_0 \left\{ 1 + \frac{\pi\rho_0 D^2}{V_0} \left[\frac{1}{4} - (x/D)^2 \right] \right\}, \quad (6)$$

integration of Poisson's equation will give us an expression for the potential which is accurate to terms of the order ρ_0^2 .

$$\begin{aligned} \frac{d^2V}{dx^2} &= 4\pi\rho_0 \left\{ 1 + \frac{\pi D^2 \rho_0}{V_0} \left[\frac{1}{4} + \left(\frac{x}{D}\right)^2 \right] \right\}, \\ \frac{dV}{dx} &= 4\pi\rho_0 \left\{ x + \frac{\pi D^3 \rho_0}{V_0} \left[\frac{1}{4} \left(\frac{x}{D}\right) - \frac{1}{3} \left(\frac{x}{D}\right)^3 \right] \right\}, \\ V &= 4\pi\rho_0 \left\{ \frac{x^2}{2} + \frac{\pi D^4 \rho_0}{V_0} \left[\frac{1}{8} \left(\frac{x}{D}\right)^2 - \frac{1}{12} \left(\frac{x}{D}\right)^4 \right] \right\} + C_4. \end{aligned}$$

When $x = D/2$ $V = V_0$,

$$V_0 = 4\pi\rho_0 \left\{ \frac{D^2}{8} + \frac{\pi D^4 \rho_0}{V_0} \left[\frac{1}{32} - \frac{1}{12 \times 16} \right] \right\} + C_4,$$

$$C_4 = V_0 - \frac{\pi\rho_0 D^2}{2} - \frac{5\pi^2 D^4 \rho_0^2}{48 V_0},$$

$$\begin{aligned} V &= V_0 - 2\pi\rho_0 D^2 \left[\frac{1}{4} - \left(\frac{x}{D}\right)^2 \right] \\ &\quad - \frac{\pi^2 \rho_0^2 D^4}{V_0} \left[\frac{5}{48} - \frac{1}{2} \left(\frac{x}{D}\right)^2 + \frac{1}{3} \left(\frac{x}{D}\right)^4 \right]. \end{aligned}$$

Repeated application of this procedure gives the following result:

$$\begin{aligned} \rho &\approx \rho_0 \left\{ 1 + \frac{\pi\rho_0 D^2}{V_0} \left[\frac{1}{4} - \left(\frac{x}{D}\right)^2 \right] + \frac{\pi^2 \rho_0^2 D^4}{V_0^2} \right. \\ &\quad \times \left[\frac{7}{48} - \left(\frac{x}{D}\right)^2 + \frac{5}{3} \left(\frac{x}{D}\right)^4 \right] + \frac{\pi^3 \rho_0^3 D^6}{V_0^3} \\ &\quad \times \left[\frac{122}{1152} - \frac{92}{96} \left(\frac{x}{D}\right)^2 + \frac{35}{12} \left(\frac{x}{D}\right)^4 - \frac{28}{9} \left(\frac{x}{D}\right)^6 \right] \\ &\quad + \frac{\pi^4 \rho_0^4 D^8}{V_0^4} \left[\frac{197}{2304} - \frac{133}{144} \left(\frac{x}{D}\right)^2 + \frac{35}{9} \left(\frac{x}{D}\right)^4 \right. \\ &\quad \left. \left. - \frac{70}{9} \left(\frac{x}{D}\right)^6 + \frac{55}{9} \left(\frac{x}{D}\right)^8 \right] \right\}. \end{aligned} \quad (7)$$

Substitution of this value of ρ into Poisson's equation and integration twice with respect to x gives the following expression for the potential:

$$\begin{aligned}
 V \simeq & V_0 \left\{ 1 - \frac{\delta^2}{9} \left[\frac{1}{4} - \left(\frac{x}{D} \right)^2 \right] \right. \\
 & - \delta^2 (4/9)^2 \left[\frac{5}{48} - \frac{1}{2} \left(\frac{x}{D} \right)^2 + \frac{1}{3} \left(\frac{x}{D} \right)^4 \right] \\
 & - \delta^3 (4/9)^3 \left[\frac{1}{18} - \frac{7}{24} \left(\frac{x}{D} \right)^2 + \frac{1}{3} \left(\frac{x}{D} \right)^4 - \frac{2}{9} \left(\frac{x}{D} \right)^6 \right] \\
 & - \delta^4 (4/9)^4 \left[\frac{11}{288} - \frac{61}{288} \left(\frac{x}{D} \right)^2 + \frac{23}{72} \left(\frac{x}{D} \right)^4 \right. \\
 & \left. - \frac{7}{18} \left(\frac{x}{D} \right)^6 + \frac{2}{9} \left(\frac{x}{D} \right)^8 \right] - \delta^5 (4/9)^5 \\
 & \times \left[\frac{323}{10368} - \frac{197}{1152} \left(\frac{x}{D} \right)^2 + \frac{133}{432} \left(\frac{x}{D} \right)^4 \right. \\
 & \left. - \frac{14}{27} \left(\frac{x}{D} \right)^6 + \frac{5}{9} \left(\frac{x}{D} \right)^8 + \frac{22}{81} \left(\frac{x}{D} \right)^{10} \right]. \quad (8)
 \end{aligned}$$

These expressions are not exact because the higher order terms in ρ_0 have not been carried in the calculation. The procedure may be used to give as many terms as desired of an infinite series for the potential distribution. If the equations just given are treated as closed expressions, they give results accurate to better than one percent for $0 \leq \delta \leq 1$. To approximate results in the range $1 \leq \delta \leq 2$, it is desirable to introduce correction terms. The terms given below are chosen to approximate the solution⁶ in closed form without the necessity of a large number of terms. The terms which added to the right-hand member of Eq. (8) give the behavior of the potential up to $\delta = 2$ are

$$\begin{aligned}
 \Delta V = & -\delta^8 (4/9)^8 0.02689 [1 - 5.78(x/D)^2 \\
 & + 7.12(x/D)^4] - \delta^7 (4/9)^7 0.0239 \\
 & \times [1 - 5.593(x/D)^2 + 6.372(x/D)^4] \\
 & - \delta^{10} (4/9)^{10} 0.025 [1 - 5.493(x/D)^2 \\
 & + 5.966(x/D)^4] - \delta^{20} (4/9)^{20} 1.250 \\
 & \times [1 - 7.366(x/D)^2 + 13.344(x/D)^4]. \quad (9)
 \end{aligned}$$

Equations (8) and (9) give the leading terms of the infinite series expressing the potential in terms of the current density. Application of the ratio test for convergence to the terms of this series suggests that it will converge up to and including the value $\delta = 2$. To the series resulting when Eq. (8) is differentiated with respect to δ term by term the ratio test may also be applied. The test suggests that the derived series converges for $\delta < 2$ and diverges for $\delta \geq 2$. Solutions in closed form⁸ confirm the conclusion that the maximum current that can be transmitted between the electrodes is twice the space-charge limited value. The difference between the two limiting cases is that for the current density $2J_0$, the potential minimum between the plates is $\frac{1}{4}V_0$ and that for the current density J_0 , the potential at the minimum is zero.

We wish to emphasize that if the current is small, the first two terms of the series [Eq. (8)] give an adequate approximation to the potential and that assuming this distribution of potential for use in place of V_0 in Eq. (2), the integration of Poisson's equation at once gives an expression with the first three terms of the series shown on the right-hand side. These are simple problems which illustrate the application of Poisson's equation to the determination of the potential distribution in a practical case.

Although the terms given here for the determination of the potential distribution have been simplified by approximation of the effect of the higher terms of the series, the terms remaining may seem cumbersome. To derive the dependence of potential on distance by this method does involve this sequence of polynomials with increasing numbers of terms. A point of particular interest, however, is the value of the potential corresponding to the potential minimum. This may be obtained directly from Eq. (8) by dropping all those terms containing x . The resulting series is not difficult to use. It is in fact far easier to obtain such information from this treatment than from solutions of the problem in closed form by the use of the trial-and-error methods.

In the present paper we should like to carry the interpretation of the result a little further to obtain additional relationships with physical significance. We represent by Q the charge con-

tained in a rectangular prism of unit cross-sectional area and height corresponding to the separation of the electrode planes, and we represent the transit time of an electron between the planes by T . Then, in general,

$$Q = JT. \quad (10)$$

We differentiate this equation with respect to J and evaluate the derivatives for the condition $J=0$.

$$\frac{dQ}{dJ} = T + J \frac{dT}{dJ} \quad (11)$$

$$\frac{d^2Q}{dJ^2} = 2 \frac{dT}{dJ} + J \frac{d^2T}{dJ^2}. \quad (12)$$

In general, therefore, when $J=0$,

$$\left(\frac{d^n Q}{dJ^n} \right)_0 = n \frac{d^{n-1} T}{dJ^{n-1}}. \quad (13)$$

Thus the Maclaurin's series expansion for the charge contained between the planes in terms of powers of the current density J is

$$\begin{aligned} Q &= \left(\frac{dQ}{dJ} \right)_0 J + \frac{1}{2} \left(\frac{d^2Q}{dJ^2} \right)_0 J^2 + \frac{1}{6} \left(\frac{d^3Q}{dJ^3} \right)_0 J^3 \\ &\quad + \cdots + \frac{1}{n!} \left(\frac{d^n Q}{dJ^n} \right)_0 J^n + \cdots \\ &= T_0 J + \left(\frac{dT}{dJ} \right)_0 J^2 + \frac{1}{2} \left(\frac{d^2T}{dJ^2} \right)_0 J^3 \\ &\quad + \cdots + \frac{1}{(n-1)!} \left(\frac{d^{n-1} T}{dJ^{n-1}} \right)_0 J^n + \cdots \quad (14) \end{aligned}$$

Now, since the series developed in the first part of this paper are in powers of the current ratio $\delta \equiv J/J_s$, the series provide us with a means of evaluating these derivatives of the transit time. First by elimination of one of the integrations with respect to x or by differentiation of Eq. (8) with respect to x we obtain an expression for the field. This is then evaluated at $x=D/2$ or $x=-D/2$. The difference of these two fields, or twice the absolute value of either result, is according to Gauss' law just $4\pi Q$. Comparison of this result with the series for Q in terms of the

transit time establishes the following results:

$$T_0 = \frac{D}{[V_0(2e/m)]^{\frac{1}{2}}},$$

$$\left(\frac{dT}{dJ} \right)_0 = \frac{\pi D^3 m}{12 V_0^2 e},$$

$$\left(\frac{d^2T}{dJ^2} \right)_0 = \frac{\pi^2 D^5}{6(2e/m)^{\frac{1}{2}} V_0^{\frac{7}{2}}},$$

$$\left(\frac{d^3T}{dJ^3} \right)_0 = \frac{\pi^3 D^7}{12 V_0^5 (e/m)^2}.$$

The first of these relationships is well known. The others relate the behavior of the transit time to the geometry, the energy of the electrons, and the properties of the electron. They enable us by means of a Maclaurin's series to relate the transit time to the current.

$$\begin{aligned} T &= \frac{D}{[V_0(2e/m)]^{\frac{1}{2}}} + \frac{\pi D^3 m J}{12 V_0^2 e} \\ &\quad + \frac{\pi^2 D^5 J^2}{12(2e/m)^{\frac{1}{2}} V_0^{\frac{7}{2}}} + \frac{\pi^3 D^7 J^3}{72 V_0^5 (e/m)^2} + \cdots \quad (15) \end{aligned}$$

INDICATED EXTENSION OF THE METHOD TO PLANES AT DIFFERENT POTENTIALS

Although solving the general problem in which the bounding planes differ in potential would carry us beyond the scope of this paper, it may be well to illustrate the generality of the method by obtaining the first term of this solution involving the current. The resulting expression will be immediately applicable to those cases in which the space charge is too small to establish a potential minimum between the planes.

Let the potential in the plane of the screen grid be represented by V_s and similarly the potential of the plate by V_p . Let the spacing between these planes, as before, be represented by D . Represent the potential gradient at the plane of the screen by $(dV/dx)_s$. In the absence of any space charge the potential distribution between the grid and the plate is given by

$$V_0(x) = V_s + (V_p - V_s)x/D. \quad (16)$$

Therefore, for small currents the space charge density will be given approximately by

$$\rho = J \{ (2e/m) (V_s + V_p x/D - V_s x/D) \}^{-\frac{1}{2}}. \quad (17)$$

The use of Poisson's equation gives for the potential, approximately,

$$V(x) \approx V_s + (dV/dx)_s x + 4\pi J (2e/m)^{-\frac{1}{2}}$$

$$\begin{aligned} & \times \int_0^x dx \int_0^x \frac{dx}{\left[V_s + \frac{(V_p - V_s)x}{D} \right]^{\frac{1}{2}}} \\ & = V_s + (dV/dx)_s x + 4\pi J (m/2e)^{\frac{1}{2}} \\ & \quad \times \int_0^x dx \frac{D^2}{(V_p - V_s)} \\ & \quad \times \left[\left\{ V_s + \frac{(V_p - V_s)x}{D} \right\}^{\frac{1}{2}} - (V_s)^{\frac{1}{2}} \right] \\ & = V_s + (dV/dx)_s x + \frac{8\pi J D^2 (m/2e)^{\frac{1}{2}}}{(V_p - V_s)^2} \\ & \quad \times \left\{ \frac{2}{3} \left[V_s + \frac{(V_p - V_s)x}{D} \right]^{\frac{3}{2}} - \frac{2}{3} V_s^{\frac{3}{2}} \right. \\ & \quad \left. - \frac{(V_p - V_s)}{D} x (V_s)^{\frac{1}{2}} \right\}. \quad (18) \end{aligned}$$

To satisfy the boundary condition at the plate,

$$\begin{aligned} \left(\frac{dV}{dx} \right)_s &= \frac{V_p - V_s}{D} - \frac{8\pi J D (m/2e)^{\frac{1}{2}}}{(V_p - V_s)^2} \\ &\times \left[\frac{2}{3} V_p^{\frac{3}{2}} - (V_p - V_s) (V_s)^{\frac{1}{2}} - \frac{2}{3} V_s^{\frac{3}{2}} \right]. \quad (19) \end{aligned}$$

Through the insertion of Eq. (19) into Eq. (18) we have an approximate expression for the potential as a function of the distance in terms

of the electrode potentials, the separation between the planes, and the current density.

CONCLUSION

Starting with the assumption of uniform distribution of charge, we have derived the potential distribution between parallel planes. Then, considering the effect of this variation of potential on the charge density, we have successively corrected our expression by additional terms. If a small fraction of space-charge-limited current density passes ($\delta \leq 0.2$), this analysis shows that the first term gives a useful approximation sufficiently accurate for many purposes. The importance of this problem has led us to calculate sufficient terms to approximate the potential distribution over the entire range of currents given by $0 \leq \delta \leq 2$, which interest us.

Fundamentally, we have derived our series by considering in detail the variation of transit time with current density. As a summary of these effects we have been able to evaluate the leading terms of the Maclaurin's series expressing the transit time as a function of current.

A comparison of our results with those of Copeland and Eggenberger reveals that our series converge very much less rapidly. The reason for this is interesting. In the case emphasized by Copeland and Eggenberger the electrons started from rest at the cathode of the region in which the potential distribution was analyzed. The important contribution of the extremely low velocity in the neighborhood of the cathode was always present regardless of the current density. In our problem, on the other hand, the current density is the primary cause of the reduction of velocity in region under study. This results in important contributions to the potential distribution from the higher order terms.

Tape Recordings of Papers Presented at the January, 1954, Annual Meeting

Tape recordings of symposia, invited papers, and some contributed papers given at the Annual Meeting of AAPT in New York on January 28, 29, and 30, 1954, may be obtained by writing to PROFESSOR J. G. WINANS, University of Wisconsin, Madison, Wisconsin.

Father Procopius Diviš and His Lightning Conductor

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(Received September 14, 1953)

A description of a grounded lightning conductor erected in 1754 in Přímetice, Moravia.

JUST as many physicists today concentrate their attention on one or two especially popular fields such as atomic or nuclear research, so in the time of man's early exposure to electrical phenomena many independent explorers experimented in various aspects of that pioneering field. While Benjamin Franklin was having his lively correspondence with Peter Collinson of the Royal Society of London primarily on his experimentation in the field of electricity in various parts of the Continent, particularly in

France and Germany, electrical experiments were being performed that met with wide public interest in that enlightened and promising 17th century. Besides the popular displays of the French electricians, Dalibard and Delor, the European public was greatly stirred by the circumstances of the tragic end, in 1753, of Georg W. Richmann of the St. Petersburg Imperial Academy of Science. He was killed in his laboratory while experimenting with a lightning rod, probably ungrounded, as was Dalibard's the previous year in France.

Another early zealous experimenter in electricity, Father Proprius Diviš, of Moravia, also reacted promptly to the fatal accident of Professor Richmann by accelerating work on his own lightning conductor. Electricity was then a widely discussed topic in Europe as was the novelty of the revolutionary studies of the nature of atmospheric electricity in which this Moravian priest, Father Procopius Diviš, had for many years delved. Details of the work of Diviš are given in a recent paper published in *Isis*.¹ (See Fig. 1.)

The purpose of this article is to reproduce and comment upon the plates showing the complexity of Diviš' lightning conductor and also the title page of Diviš' treatise on electricity, which was first published in 1765, in Tübingen. The appearance of the Diviš lightning conductor very well indicates his conviction in the ability of points to "draw off" the electrical fire from the clouds. This reaction of metallic points features very strongly in Franklin's ideas and once became the topic of heated discussion when involved in political implications. Thus, the Diviš lightning conductor, with its large number of points, was to serve as an instrument intended to prevent the accumulation of atmospheric electricity and consequently the formation of lightning. Indeed,



FIG. 1. Father Procopius Diviš, born March 26, 1698, in Zamberg, Bohemia; Died December 21, 1765, in Přímetice, Moravia. Reproduction of an original engraving. In his *Geschichte der Physik*, J. C. Poggendorff maintains that Diviš erected the first complete lightning conductor in Europe.

¹ K. Hujer, *Isis*, 43, 351 (1952).

as we note from the plate, there were twelve metallic boxes of 33 points each, with the thirteenth and larger box containing approximately 77 points. On the basis of his many experiments with points, also performed at the Imperial Court in Vienna in 1750, the Moravian

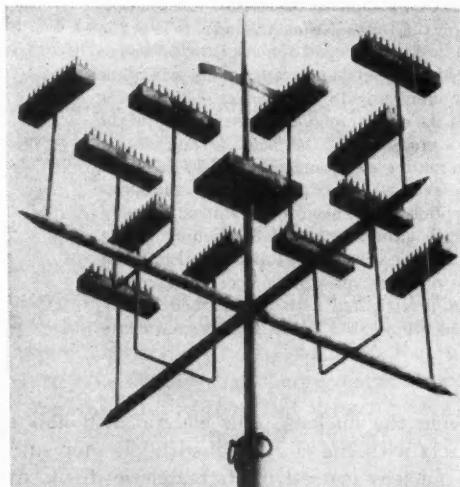


FIG. 2. A view of the top of the Diviš lightning conductor. It was grounded by means of chains attached to rings at the bottom of the vertical arm. Reproduction according to the original description in the Diviš manuscript on the Meteorological Machine. The Diviš lightning conductor was erected in the parish garden in Přímětice, Moravia, June 15, 1754, where it stood six years. The large number of points indicates the principle of Diviš lightning conductor: the power of points "to draw off" the electrical "fire" from charged bodies.

priest assumed that the large number of points would suck the electrical fluid from the atmosphere as soon as it was produced. (See Fig. 2.)

The further original feature of this conductor is in its grounding. It apparently was the first grounded lightning conductor to appear in

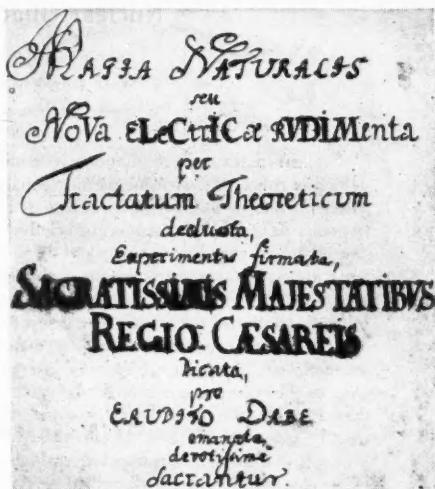


FIG. 3. Title page of the manuscript of Diviš' principal work *Magia Naturalis* in the Olomouc Museum Library, Olomouc, Moravia. This work, under the title *Theoretischer Tractat, oder die längst verlangte Theorie von der meteorologischen Electricität* was published in German translation by J. Schramm in Tübingen, 1765. It reached its second edition in Frankfurt by 1768. Some five copies of this publication are known to exist today.

Europe after the fateful experience of Richmann. Dalibard, Delor, and Richmann tried to prove that the nature of lightning is electrical, which was also Franklin's theory. Diviš applied this verified theory and on June 15, 1754, erected in his parish garden in Přímětice, Moravia, his lightning conductor, which, according to J. C. Poggendorff's *Geschichte der Physik*, is apparently the first lightning conductor in its complete form erected in Europe. We have no evidence that Diviš had ever heard of Franklin. This would indicate how various experiments, performed independently yet gravitating about the same subject and scope, usually arrive at similar ends and results. (See Fig. 3.)

Electronic Equipment

A Symposium on the Automatic Production of Electronic Equipment will be held at San Francisco's Fairmont Hotel April 19-20, 1954. Joint sponsors are Stanford Research Institute and the United States Air Force. General chairman of the symposium is L. K. LEE, head of the Advanced Techniques Group, Engineering Division, Stanford Research Institute.

Nuclear Quadrupole Resonance*†

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(Received July 10, 1953)

Nuclear quadrupole resonance is a recently developed branch of radio-frequency spectroscopy which is concerned with magnetic resonance absorption in crystals. This absorption is due to reorientation of the nonspherical atomic nuclei against crystalline electric fields. Related phenomena in isolated molecules are briefly mentioned in the Introduction A. This is followed in Sec. B by an elementary treatment of the electrostatic interaction of a nonspherical nucleus with an axially symmetric inhomogeneous electric field as is found approximately in crystalline Cl_2 , a crystal whose structural units are very nearly undisturbed Cl_2 molecules. Electrostatic torques are shown to exist which cause a precession of the nuclear angular momenta around the molecular axes. As this motion is accompanied by a precession of the magnetic and electric moments of the nuclei, which may interact with alternating electromagnetic fields, this leads over to the transition mechanism between the energy levels which are obtained from the corresponding Hamiltonian. Transition and power absorption phenomena are discussed in more detail in Sec. C. In Sec. D effects are treated which either lead to a broadening of the absorption lines or are connected with the thermal agitation in the crystalline lattice. Some experimental aspects are described and examples of observed absorption lines are given in Sec. E. In the concluding Sec. F all nuclei so far investigated with this method are reviewed in three tables. The nuclear information obtainable from such experiments is pointed out.

A. INTRODUCTION

NUCLEAR quadrupole resonance,¹ a recently developed^{2,3} branch of radio-frequency spectroscopy, has already contributed significant information on nuclear quadrupole moments as well as molecular and crystalline structure. Experimentally it is concerned with the detection of *radio-frequency magnetic resonance absorption* in suitable (molecular) crystals. In this and many other respects it is similar to nuclear magnetic resonance.^{4,5} However, while in nuclear magnetic resonance one has to do with transitions between levels corresponding to different orientations of the nuclear magnetic moments against a static magnetic field applied from the outside, no such field is needed in NQR. The place of the magnetic field is taken by an inhomogeneous (axially symmetric) electric field generated by the charge cloud of the (diatomic) molecule con-

taining the nucleus. This electric field now interacts with the electric quadrupole moment of the nucleus instead of its magnetic dipole moment. The quadrupole moment measures the deviation from spherical symmetry of the nuclear charge distribution. Again, different relative orientations of nuclear axis against field axis correspond to different energy levels. For free molecules such nuclear quadrupole effects have been observed earlier in molecular beam resonance experiments⁶ and in rotational absorption spectra in the microwave region,⁷ where they lead to a hyperfine structure of the lines.

B. MOTION AND ENERGY EIGENVALUES IN A QUADRUPOLE SYSTEM*

We now consider in more detail a system which consists of a nonspherical nucleus in a diatomic molecule forming part of a molecular crystal, to be specific, one Cl nucleus in solid Cl_2 . In such a crystal the molecules essentially retain their identity and the electrostatic field generated at

* This article is an enlarged version of an invited paper given at the Cambridge Meeting of the American Physical Society in January 1953.

† Supported by the Office of Ordnance Research.

‡ Henceforth abbreviated NQR.

¹ H. G. Dehmelt and H. Krüger, *Naturwiss.* **37**, 111 (1950); *Z. Physik* **129**, 401 (1951).

² R. V. Pound, *Phys. Rev.* **79**, 685 (1950).

³ Bloembergen, Purcell, and Pound, *Phys. Rev.* **73**, 679 (1948).

⁴ F. Bloch, *Phys. Rev.* **70**, 460 (1946); Bloch, Hansen, and Packard, *Phys. Rev.* **70**, 474 (1946).

⁵ Kellogg, Rabi, Ramsey, and Zacharias, *Phys. Rev.* **55**, 318 (1939); **57**, 677 (1940).

⁶ D. K. Coles and W. E. Good, *Phys. Rev.* **70**, 979 (1946).

⁷ Compare H. B. G. Casimir, *On the Interaction between Atomic Nuclei and Electrons* (Teyler's Tweede Genootschap, E. F. Bohn, Haarlem, 1936); *Physica* **2**, 719 (1935).

the site of the Cl nucleus is practically determined by the molecular electron cloud alone. Therefore it is very nearly axially symmetric, even though the surroundings in the lattice have lower symmetry. We introduce a frame of reference xyz fixed in space (see Fig. 1), the z axis coinciding with the molecular axis and in the point of origin the Cl nucleus under consideration, which in Figs. 1 and 2 is represented as an highly enlarged ellipsoid. Further, a second frame x', y', z' , with z' fixed along the nuclear axis is chosen. Since both nucleus and surrounding molecular field q are axially symmetric, the electrostatic interaction energy

$$W = \int \rho_N q dV \quad (B1)$$

can only depend on the angle θ between nuclear and molecular axis. To simplify the calculation we therefore let the y and y' axes fall together. Quantity ρ_N denotes the charge distribution in the nucleus. If the potential q as function of $x'y'z'$ is expanded around the origin, W takes the form

$$\begin{aligned} W = & q(0) \int \rho_N dV' + q_{z'}(0) \int \rho_N x' dV' + \dots \\ & + \frac{1}{2} \left[q_{z'z'}(0) \int x'^2 \rho_N dV' + q_{y'y'}(0) \right. \\ & \times \left. \int \rho_N y'^2 dV' + q_{z'z'}(0) \int \rho_N z'^2 dV' \right] + \dots \quad (B2) \end{aligned}$$

The first constant term in this expression is of no interest to us. The following (dipole) terms containing the first derivatives of q vanish, as the integrals are zero because of the symmetry of the nucleus. The (quadrupole) terms with the second derivatives of q are the ones of interest, while higher terms can be neglected in excellent approximation as proved by the experiments. Since the integrals $\int \rho_N x'^2 dV' \dots$ constitute the components of a tensor—henceforth abbreviated $eQ_{z'z'}$, $eQ_{y'y'} \dots$ whose principal axes coincide with x' , y' , z' , no mixed terms^{8a} $Q_{x'y'}$, $Q_{x'z'}$, $Q_{y'z'}$ appear in formula (B2). We now make use of the

^{8a} Without referring to their tensor character, it can also be seen simply from the symmetry of the nucleus, that these integrals $\int \rho_N x'y'dV' \dots$ vanish.

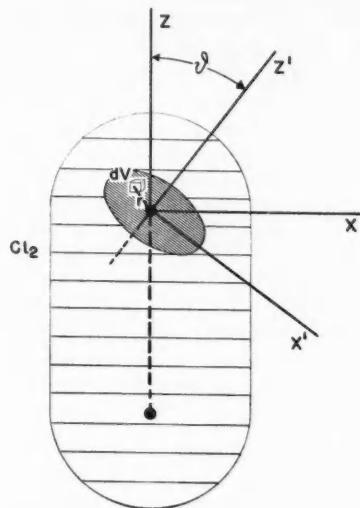


FIG. 1. Nonspherical Cl nucleus in Cl_2 molecule.

Laplace equation $q_{z'z'} + q_{y'y'} + q_{z'z'} = 0$ holding at the site of the nucleus. With $Q_{z'z'} = Q_{y'y'}$ because of the axial symmetry we obtain for the quadrupole interaction energy:

$$W_Q = \frac{1}{2} eQ_{z'z'} (Q_{z'z'} - Q_{z'z'}). \quad (B3)$$

This reduces to

$$W_Q = \frac{1}{4} q_{z'z'} \int \rho_N (3z'^2 - r^2) dV', \quad (B4)$$

since

$$\int \rho_N x'^2 dV' = \frac{1}{2} \int \rho_N (r^2 - z'^2) dV'.$$

The integral $\int \rho_N (3z'^2 - r^2) dV'$ defines an inherent property of the nucleus and is customarily called "the" nuclear quadrupole moment and denoted by eQ . It measures the deviation from spherical symmetry of the nuclear charge cloud. A nucleus elongated with respect to its axis has a positive, a flattened nucleus a negative, quadrupole moment. For a spherical nucleus eQ vanishes of course. Quantity $q_{z'z'}$ in Eq. (B4) can be expressed as

$$q_{z'z'} = q_{zz} \sin^2 \theta + q_{zz} \cos^2 \theta, \quad (B5)$$

with $q_{zz} = q_{yy} = -\frac{1}{2} q_{zz}$ this reduces to

$$q_{z'z'} = q_{zz} \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right). \quad (B6)$$

Here q_{zz} represents an inherent molecular quantity which, because of the axial symmetry, completely suffices to describe the electric field at the site of the nucleus. Finally we have obtained for the classical quadrupole interaction energy the formula:

$$W_Q = \frac{1}{4}eQq_{zz}(\frac{3}{2}\cos^2\theta - \frac{1}{2}). \quad (\text{B7})$$

Because of the dependence of the energy W_Q on θ there must result a torque, which for the appropriate sign of eQq_{zz} tends to align the nuclear and the molecular axes. As in the Zeeman effect, the nuclear angular momentum I , whose direction coincides with the figure axis of the nucleus, will respond to this torque by a precession around the z axis (see Fig. 2). However, while in the Zeeman effect the precession frequency is constant, independent of θ , we find for

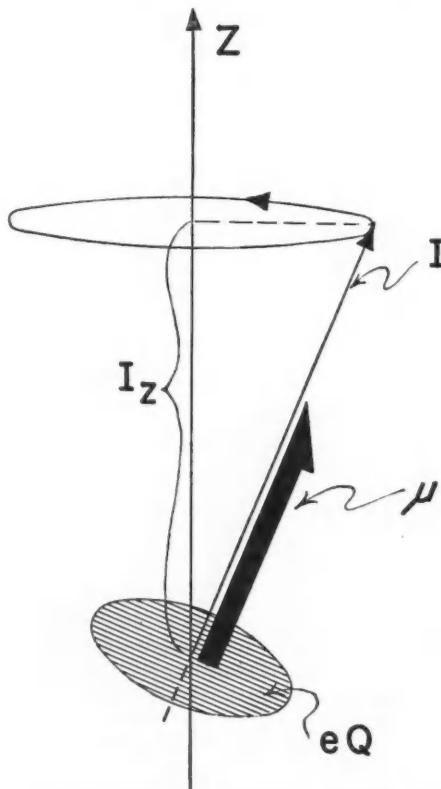


FIG. 2. Precession of nuclear moments I , μ , eQ around symmetry axis of molecular electric field.

the quadrupole system a precession which slows down with growing θ comes to a standstill for $\theta=\pi/2$, reverses its sense and between $\theta=\pi/2$ and π again increases in frequency.⁹ This precession of the angular momentum—compare Fig. 2—will also be accompanied by rotating components of the magnetic dipole μ , as well as the electric quadrupole moment eQ of the nucleus, which in the classical picture must lead to the respective radiations. It can be seen however, that only the rotating magnetic dipole experiences any appreciable coupling with an electromagnetic radiation field of the appropriate low frequency of a few megacycles while the coupling of the electric quadrupole is many orders of magnitude smaller.

Now we shall try to derive the quantum-mechanical energy eigenvalues of our quadrupole system—abbreviated QS . We note that the z component of the angular momentum I_z is a constant of the motion—compare Fig. 2—whose eigenvalues m therefore can be used to label the energy eigenvalues of the QS . By substituting the operators I_z/I for $\cos\theta$ in the classical expression (B7) we obtain the Hamiltonian

$$H = \frac{1}{8}eQ^*q_{zz}(3I_z^2 - I^2)/I^2, \quad (\text{B8})$$

from which follow the energy eigenvalues

$$E_m = \frac{1}{8}eQ^*q_{zz}[3m^2 - I(I+1)]/I(I+1). \quad (\text{B9})$$

In Eq. (B9) Q has been marked with an asterisk, in order to show that in this formula it has been evaluated with respect to the figure axis of the nucleus. This deviates somewhat from the customary definition of the nuclear quadrupole moment Q , which is to be taken for the aligned state $m=I$ of the nucleus with respect to the axis of alignment z . Even though quantum-mechanically complete alignment is not possible, the charge distribution $\rho_{m=I}$ is axially symmetric with respect to the z axis, the axis of the electric field. This is due to the fact that $\rho_{m=I}$ represents an average of the nuclear charge distribution over the motion which has not completely ceased even for $m=I$. In this case the interaction energy can easily be written down independent of Eq. (B9) using Eq. (B4) for $\theta=0$

⁹ Quantum mechanically this is reflected in the spacing of the energy levels, which is not equidistant as in the Zeeman effect but increases with growing values of the magnetic quantum number m .

and substituting $\rho_{m=I}$ for ρ_m . Thus we obtain for the energy eigenvalue $E_{m=I}$:

$$E_{m=I} = \frac{1}{4}q_{zz} \int \rho_{m=I} (3z^2 - r^2) dV. \quad (\text{B10})$$

Here the integral is identical with the customary eQ . Comparing this with E_I from Eq. (B9) we find

$$Q^* = 2Q(I+1)/(2I-1). \quad (\text{B11})$$

Since the square of the total nuclear angular momentum is a constant of the system, we can substitute for the operator I^2 in the denominator of Eq. (B8) its eigenvalue $I(I+1)$, and using Eq. (B11) we obtain from Eqs. (B8) and (B9) the familiar expressions

$$\mathbf{H} = eQq_{zz}(3\mathbf{I}_z^2 - \mathbf{I}^2)/4(2I-1)I, \quad (\text{B12})$$

and

$$E_m = eQq_{zz}[3m^2 - I(I+1)]/4(2I-1)I. \quad (\text{B13})$$

The energy levels following from Eq. (B13) are sketched in Fig. 3 for integer and half-integer I values. For $I \leq \frac{1}{2}$ the quadrupole interaction energy vanishes. This follows from formula (B9) for $I = \frac{1}{2}$ while for $I = 0$ the nuclear charge distributing in Eq. (B10) becomes spherically symmetric which reduces the integral to zero.

In case the axial symmetry of the electric field is only approximate, an asymmetry parameter may be defined:

$$\epsilon = |(q_{zz} - q_{yy})/q_{zz}|. \quad (\text{B14})$$

This asymmetry¹⁰⁻¹² causes, as a perturbation calculation shows, a first-order splitting only of the degenerate $m = \pm 1$ levels, while all other levels experience only small second-order shifts, not affecting the $\pm m$ degeneracy. In all cases

¹⁰ From the classical interaction energy in the general case of no axial symmetry of the electric field a Hamiltonian can be obtained in the same way as Eq. (B12):

$$\mathbf{H} = [eQ/2I(2I-1)][q_{zz}\mathbf{I}_z^2 + q_{yy}\mathbf{I}_y^2 + q_{zz}\mathbf{I}_y^2].$$

For this—using the definition for ϵ —can be written

$$\mathbf{H} = [eQq_{zz}/4I(2I-1)][3\mathbf{I}_z^2 - \mathbf{I}^2] + \epsilon[eQq_{zz}/4I(2I-1)][\mathbf{I}_z^2 - \mathbf{I}_y^2]$$

and the second term then treated as a perturbation.

¹¹ H. Krüger, Z. Physik **130**, 371 (1951).

^{11a} R. Bersohn, J. Chem. Phys. **20**, 1505 (1952).

^{11b} For $I = 3/2$ the small dependence of the Zeeman effect on the asymmetry of the electric field can be used to determine ϵ ; compare C. Dean, Phys. Rev. **86**, 607 (1952).

¹² Compare Bloembergen, Purcell, and Pound, Phys. Rev. **73**, 679 (1948).

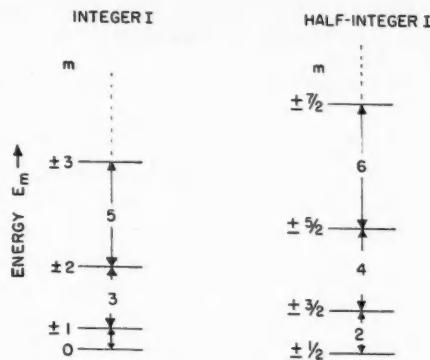


FIG. 3. Energy levels arising from the interaction of a quadrupole nucleus with an axially symmetric electric field. The diagram is drawn for $eQq_{zz} > 0$, the unit of energy being $3eQq_{zz}/4I(2I-1)$.

except $I = \frac{1}{2}$, ϵ can be determined from the observed spectrum.¹³

If we subject the QS to a static magnetic field a Zeeman effect^{11,11a} results due to interaction with the nuclear magnetic moment. The pattern of this Zeeman effect depends strongly upon the relative orientation of the electric field axis and the magnetic field. Formula (B13) has to be supplemented by an additional term

$$\mathbf{H}_{\text{mag}} = -\mu(\mathbf{I}_z H_z + \mathbf{I}_y H_y + \mathbf{I}_x H_x)/I. \quad (\text{B15})$$

If the magnetic field is parallel to the electric axis, the magnetic contributions to the energy levels given by Eq. (B13) can be written down at once, as \mathbf{I}_z is diagonal with respect to the eigenfunctions of the QS as

$$\Delta E_m = -\mu H_z m/I \quad (\text{B16})$$

removing the $\pm m$ degeneracy. For the magnetic field perpendicular to the electric axis the first-order perturbation energy vanishes for all terms other than $m = \pm \frac{1}{2}$. A perturbation calculation shows that the perturbation mixes both states $m = +\frac{1}{2}$ and $m = -\frac{1}{2}$ forming two new states 1, 2, the shifts for which are given below:

$$\Delta E_{1,2} = \pm (I + \frac{1}{2})\mu H/2I. \quad (\text{B17})$$

This is larger by the factor $I + \frac{1}{2}$ than for the magnetic field parallel to the z axis.

¹³ See, e.g., Eyring, Walter, and Kimball, *Quantum Chemistry* (John Wiley and Sons, Inc., New York 1949), Sec. 8c.

C. TRANSITIONS, RF SUSCEPTIBILITY AND POWER SATURATION¹²

From optical spectroscopy the formula¹³

$$W_{m \leftrightarrow m'} = (8\pi^2/3h^2) |(m|\mathbf{M}|m')|^2 \rho_r \quad (C1)$$

for the probability of a dipole transition per unit time from a state m into a state m' or vice versa is familiar. Symbol \mathbf{M} represents the (magnetic) moment operator and ρ_r the energy density of the radiation field causing the transition. Usually this formula is derived for the experimental conditions A.

Conditions	A	B
Polarization of radiation	isotropic	linearly polarized
Orientation of (quadrupole) systems	unpolarized z direction	isotropic (in a polycrystalline substance)
Width of energy levels	zero	finite
Spectrum of radiation	continuous	monochromatic

For our quadrupole system the nuclear magnetic moment is parallel to the angular momentum I , and so we can write

$$|(m|\mathbf{M}|m')|^2 = (\mu^2/I^2) |(m|\mathbf{I}|m')|^2. \quad (C2)$$

Now $|(m|\mathbf{I}|m')|^2$ can be expressed as the sum of the squares of the matrix elements of the vector components \mathbf{I}_x , \mathbf{I}_y , \mathbf{I}_z . As the transition elements for \mathbf{I}_x and \mathbf{I}_y are equal because of the axial symmetry while those for \mathbf{I}_z vanish, we obtain

$$|(m|\mathbf{M}|m')|^2 = (\mu^2/2I^2)(I(I+1) - mm') \quad (C3)$$

$$|m-m'| = 1.$$

Combining Eqs. (C1) and (C3) we get

$$W_{m \leftrightarrow m'} = (4\pi^2\mu^2/3h^2I^2)(I(I+1) - mm')\rho_r. \quad (C4)$$

Equation (C4) also holds for the experimental conditions B which are most frequently encountered in NQR. The effective radiation density ρ_r for case B is obtained by spreading the energy contained in the monochromatic rf radiation at frequency ν' over a spectrum the shape of which is the mirror image of that of the actually observed absorption line at frequency ν_0 which is broadened by effects to be discussed later, and treating the transition frequency as infinitely sharp instead. We choose the shape

function g of the absorption line as normalized:

$$\int_0^{+\infty} g(\nu - \nu_0) d\nu = 1, \quad (C5)$$

where ν_0 denotes the frequency of maximum absorption. If $H_1 = H_{10} \sin 2\pi\nu't$ is the magnetic rf field in the sample, the equivalent spectral energy density in formula (C4) must fulfill

$$\int_0^{+\infty} \rho_r d\nu = H_{10}^2/8\pi, \quad (C6)$$

taking an equal amount of electric field energy into account. Further we assumed $\rho_r = \alpha g(\nu' - \nu)$, with α a proportionality factor. Because of the normalization of g it is

$$\int_0^{+\infty} \rho_r d\nu = \alpha.$$

Eventually we get

$$\rho_r = (H_{10}^2/8\pi)g(\nu' - \nu). \quad (C7)$$

In order to compute the energy absorbed from the radiation field by the QS, we must consider the population of the energy levels. We shall see later, that the QS in a crystal at ordinary temperatures have enough interaction with the thermal molecular motions in the crystal lattice to establish a Boltzmann distribution of the level population corresponding to the crystal temperature in a reasonably short time. At these temperatures $h\nu$ will be small against kT and only a small surplus n will be found in the lower of two consecutive levels

$$n = |N_m - N_{m'}| \approx N_0 h\nu/kT(2I+1). \quad (C8)$$

With respect to the detectable net absorption only this small surplus need be considered as, for equal population in both levels, absorption and stimulated emission of quanta cancel. In this formula N_0 is the number of systems in unit volume and N_m and $N_{m'}$ the number of systems in the states m , m' . Under the assumption that the radiation is weak enough not to upset the Boltzmann distribution, we can calculate the absorbed power P_a as number of transitions in unit time times energy of quantum absorbed:

$$P_a = n W_{m \leftrightarrow m'} h\nu$$

$$= W_{m \leftrightarrow m'} N_0 (h\nu)^2 / kT(2I+1). \quad (C9)$$

The apparent Q of the sample is defined as the ratio of 2π times the rf energy stored in the sample volume to the energy absorbed in the sample per cycle:

$$Q = \nu H_{10}^2 / 4P_a. \quad (C10)$$

This formula is useful in calculating the reduction of the Q of a rf coil containing the sample. Quite generally Q is connected with the imaginary part of the rf susceptibility

$$\chi'' = 1/4\pi Q. \quad (C11)$$

For simple line shapes, $g(0) = g_{\max}$ can be expressed conveniently by the line width between half-maximum points $\Delta\nu$, e.g., for the damped oscillator line shape it is

$$g(0) = 2/\pi\Delta\nu. \quad (C12)$$

Using Eqs. (C4–12) we finally write down

$$\chi'' = \frac{2}{3} \frac{N_0 \mu^2}{(2I+1)kT} \frac{I(I+1)-mm'}{I^2} \frac{\nu}{\Delta\nu}. \quad (C13)$$

The factor 2 in this formula takes into account, that both of the transitions $+m \leftrightarrow +m'$ and $-m \leftrightarrow -m'$ fall on the same frequency.^{13a} The formulas (C1)–(C13) have been obtained under the assumption of weak irradiation of the sample. For high radiation densities however it has to be taken into account, that the absorption of quanta by the QS will tend to reduce the population surplus in the lower of the two levels between which the transitions occur. Insofar as further absorption is concerned this is equivalent to a rise in the effective temperature T_s of the QS. Instead of the lattice temperature T_L now the higher T_s has to be used in formula (C13) leading to a smaller value of the rf susceptibility χ'' . The equilibrium value of T_s can be obtained by equating the rate of temperature rise due to rf radiation "heating" $(\partial T_s / \partial t)_R$ of the QS and the rate of temperature drop $-(\partial T_s / \partial t)_L$ due to cooling through the thermal contact with the crystal lattice. It can be shown¹⁴ that the heat exchange between the QS and the lattice takes place

^{13a} As those two transitions correspond to opposite senses of precession, the resulting net rf magnetization does not rotate, but pulsates, always lying in the direction of the applied rf field.

¹⁴ See Sec. VII of reference 12.

according to the law

$$(\partial T_s / \partial t)_L = (1/T_1)(T_L - T_s)(T_s/T_L), \quad (C14)$$

which, except for the factor T_s/T_L , is familiar from macroscopic conduction cooling. Symbol T_1 denotes the "spin-lattice" or "thermal" relaxation time.

On the other hand $(\partial T_s / \partial t)_R$ can be expressed using formula (C8):

$$(\partial T_s / \partial t)_R = -(T_s/n)(\partial n / \partial t).$$

Since

$$\partial n / \partial t = -2nW_{m \leftrightarrow m'},$$

we finally obtain from $(\partial T_s / \partial t)_R + (\partial T_s / \partial t)_L = 0$

$$T_s = (1 + 2T_1W_{m \leftrightarrow m'})T_L. \quad (C15)$$

A critical probability W^* may be defined as the one for which $T_s = 2T_L$, and therefore χ'' is reduced to half of its original value:

$$W^* = (1/2T_1). \quad (C16)$$

The corresponding critical rf field strength H_{10}^* can be calculated from the foregoing, when T_1 is known.

D. LINE WIDTH, TEMPERATURE DEPENDENCE OF TRANSITION FREQUENCIES, AND SPIN-LATTICE RELAXATION

Experimentally observed NQR lines show relative widths varying between 10^{-2} and 10^{-5} of the transition frequency. The broadening may be ascribed to three causes:

a. Imperfections of the crystal lattice cause the field gradient to vary slightly from molecule to molecule as it somewhat depends upon the arrangement of the neighbor molecules, which then need not be exactly the same for all molecules. Such imperfections may be caused by elastic strains in or a too fast disorderly growth of the crystal or by the formation of mixed crystals with impurities.

b. Part of the line width is always due to the magnetic dipole-dipole interaction^{15,3} of the nuclei in a rigid lattice. It can be calculated approximately by finding $\langle H_z^2 \rangle_M$, the average of the square of the component of the field along the symmetry axis generated at the site of the

¹⁵ A. Abragam and K. Kambe, Bull. Am. Phys. Soc. **28**, No. 1, 19 (1953) and Phys. Rev. **91**, 894 (1953).

quadrupole nucleus by the magnetic moments of the surrounding nuclei and looking at the resulting Zeeman effect. However, this simple procedure is only applicable in the case that the nuclei generating the broadening field have precession frequencies different from the nucleus under consideration, as, e.g., the protons in organic iodine compounds. For transitions other than $m = \pm \frac{1}{2} \leftrightarrow m = \pm \frac{3}{2}$, the line width is then approximately given by formulas (23) and (24) of reference 3. Because of the sensitivity of the $m = \pm \frac{1}{2}$ terms^{16,17} to field components perpendicular to the symmetry axis a considerably stronger broadening is observed for the $m = \pm \frac{1}{2} \leftrightarrow m = \pm \frac{3}{2}$ transition.

c. The torsional vibrations¹⁷⁻¹⁹ of the molecules in the crystal lattice modulate the field gradient and in this way also contribute to the broadening²⁰ of the lines. This type of line broadening is closely connected with the temperature dependence of the NQR frequencies which also can be understood as caused by torsional vibrations. For simplicity we assume the torsional vibration to take place in the z, x plane only. Raman data for such oscillations lead to frequencies of the order of 10^{11} to 10^{12} sec⁻¹. This is much higher than NQR frequencies usually observed and gives us the justification to average the field gradient over the torsional motion. Using formula (B6) with θ now denoting the angle between the molecular axis and the equilibrium position, we get

$$(q_{zz})_{Av} = q_{zz}(\frac{3}{2}(\cos^2\theta)_{Av} - \frac{1}{2}). \quad (D1)$$

For ordinary temperatures we can treat the thermally excited oscillations classically.

With $(D/2)(\theta^2)_{Av} = kT/2$ follows:

$$(q_{zz}(T))_{Av} \approx q_{zz}(1 - 3kT/2D).$$

This predicts a linear decrease of the transition frequencies with rising temperature T . Substituting for the torque constant D , values calculated from representative Raman data, one obtains for the temperature dependence of the transition frequencies values compatible with the experi-

mental data for

$$(1/\nu)(d\nu/dT) \approx -10^{-4} [\text{°K}^{-1}].$$

Up to now we have concerned ourselves only with the "long" time average of the torsional amplitude. In more detail this amplitude will vary statistically with a characteristic time τ which is given by the reciprocal width of the torsional oscillator resonance curve on a frequency scale, which also could be obtained from Raman spectra. As in the treatment of the nuclear relaxation times in liquids,³ the fluctuations of the field gradient averaged only over a torsional cycle can be resolved in a flat continuous frequency spectrum stretching from zero to about $1/\tau$. Then the low-frequency fluctuation components of $(q_{zz})_{Av}$ will only broaden the lines (T_2 -effect), while the spectral fluctuation components of $(q_{zz})_{Av}$ at the quadrupole transition frequencies give rise to a strong spin-lattice relaxation mechanism (T_1 effect), which also contributes to the line broadening.

E. EXPERIMENTAL ASPECTS

Exactly as in nuclear magnetic resonance (NMR), NQR absorption lines are detected by making the sample the core of a rf coil which is part of a tank circuit and displaying in some way the reduction of the Q of this circuit when it is tuned over an absorption line. However, as opposed to NMR where it is possible to keep the frequency constant and shift the energy levels by varying the static magnetic field, in NQR experiments the spectrometer *must* have a sufficiently wide-frequency range. This calls for simple and easily tunable circuits. The effects to be detected are not strong and can easily be lost in the noise generated in the tubes and other parts of the detection circuit. In a good spectrometer the noise is essentially only the thermal noise of the tank circuit containing the sample. However, at the same time the circuit should be able to develop a rf power as high as possible, since the absorbed power is proportional to the rf power incident upon the sample. While regenerative oscillators^{21,22} are quite satisfactory at low power

¹⁶ See formula (B17) of this paper.

¹⁷ L. Pauling, Phys. Rev. **36**, 430 (1930).

¹⁸ A. Eucken, Z. Elektrochem. **45**, 126 (1939).

¹⁹ A. Kastler and A. Rousset, Phys. Rev. **71**, 455 (1947).

²⁰ Detailed calculations by H. Bayer, Z. Physik **130**, 227 (1951) and Y. Ayant, C. R. Acad. Sci. (Paris) **236**, 1553 (1953).

²¹ R. V. Pound and W. D. Knight, Rev. Sci. Instr. **21**, 219 (1950).

²² Ralph Livingston, Ann. N. Y. Acad. Sci. **55**, 800 (1952).

levels, they do not allow the generation of higher rf powers without generating an excessive amount of tube noise at the same time so that nothing in signal-to-noise ratio can be gained in this way. In NMR this is not too serious, as the early onset of saturation in the small samples usually employable forbids the use of higher power anyway. A different situation is encountered in NQR where in many cases saturation can easily be avoided by the use of large samples or need not be feared because short relaxation times are to be expected due to the relaxation mechanism sketched earlier. Here the use of superregenerative oscillators^{1,23,24} has proven especially advantageous. The obvious shortcomings of these circuits are not serious as long as the experimental aim is not the exact measurement of line shapes or relaxation times, but highest sensitivity in detecting unknown lines.

The superregenerative spectrometer employs an oscillator circuit whose tube is cut off periodically by applying a large negative grid bias for half a period. The period length of this "quench" frequency is about ten times as long as the decay time constant of the tank circuit, so that the oscillations die down to a millivolt or less before the tube is switched on again. First the resulting rf pulses excite the quadrupole system. Then part of the excitation energy flows back into the tank circuit and slightly affects the minimum amplitude U_{\min} that is reached during the cut-off interval. The rf voltage which the precessing nuclei induce back into the coil during the low oscillation intervals of the quench cycles is given by $\Delta U \approx -4\pi\chi''Q'\alpha U_{\max}$, where Q' denotes the figure of merit of the tank circuit and α is a factor somewhat less than unit correcting for the pulsating character of the rf field exciting the nuclei. Now the time after which the maximum value U_{\max} of the oscillations is attained depends sensitively on the minimum amplitude from which the oscillations build up. In this way the nuclear signal causes the time average of the oscillation amplitude to change. These changes are then fed into an amplifier and, e.g., displayed on an oscilloscope as the frequency of the oscillator is swept over

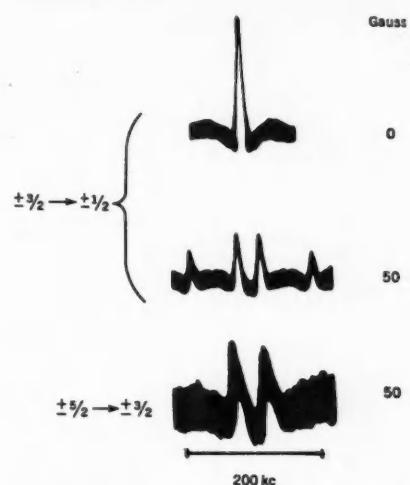


FIG. 4. NQR absorption line due to the I^{127} nuclei in SnI_4 (from table reference 10). 50 c/sec capacitive frequency modulation and oscilloscopic display was used. The two lower traces show the Zeeman pattern for both of the (occurring) transitions. The frequency scale for all three traces is the same.

the absorption line with the help of a small vibrating capacitor in parallel to the tank circuit. The line shapes depend upon the phase relation between ΔU and U_{\min} . As this relation is subject to various second-order effects in the detection circuit, usually mixtures between absorption and dispersion curves are observed *in praxi*. Absorption lines as weak as the N^{14} line near 3.3 Mc in $(CH_2)_6N_4$ have been detected using superregenerative spectrometers employing capacitive frequency modulation at 60 cycles and oscilloscopic display.

Higher sensitivity of course can be achieved by reducing the band width of the amplifying circuits in order to suppress the noise. While capacitive modulation can also be used in conjunction with narrow band recording circuits, if a few precautions are taken, it appears more satisfactory to use Zeeman modulation^{24a} of the nuclei. With this type of modulation the lines in a polycrystalline sample are periodically smeared out by zero based field pulses of about 100 gauss. No previous knowledge of the line width is necessary to obtain optimum modulation efficiency, and response to spurious signals is greatly reduced also.

²³ H. G. Dehmelt, Z. Physik 130, 385 (1951).

²⁴ For a general treatment see: J. R. Whitehead, *Super-regenerative Receivers* (Cambridge University Press, Cambridge, 1950).

^{24a} Compare table references 1 and 2.

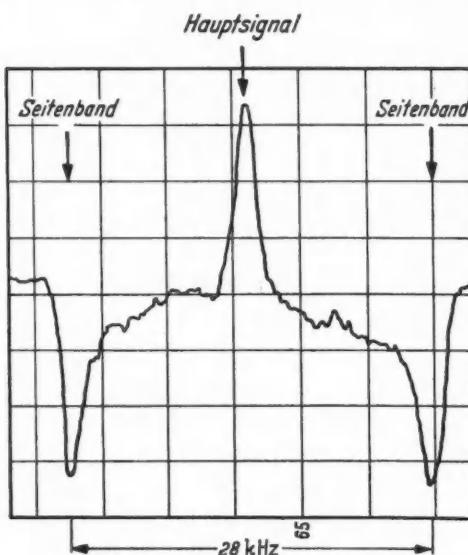


FIG. 5. Recording of the NQR absorption line in polycrystalline $B(CH_3)_3$ at 2589.0 Kc from table reference 1. The line corresponds to the transitions $m = \pm 2 \leftrightarrow \pm 3$ of the B^{10} isotope. 30 c/sec Zeeman modulation and a lock-in amplifier employing a time constant of 40 sec were used.

Without unduly long searching times one can obtain a gain in signal-to-noise ratio of about hundred by using a chart recorder instead of an oscilloscope and 60-cycle modulation to view the lines. Recently the NQR spectrum of S^{33} in a sample of rhombic sulfur containing this isotope in its natural concentration of 0.74 percent was recorded with a superregenerative spectrometer employing 30-cycle Zeeman modulation.

F. REVIEW OF INVESTIGATED NUCLEI AND NUCLEAR INFORMATION OBTAINABLE FROM NQR EXPERIMENTS

All nuclei so far investigated with the NQR method are listed in Table I. Though some nuclei have been investigated in numerous molecules, the coupling constant is given only for one or two representative molecules.

As frequencies in the rf region can easily be measured with accuracies of 10^{-5} or better, quadrupole coupling constants $eQ_{q_{zz}}$ can be obtained from NQR spectra with the same accuracy. Full use of this accuracy can be made in the determination of the ratios of quadrupole

moments of isotopic pairs.²⁵ Careful investigation of the ratio²⁶ of the coupling constants of the $Cl^{35,37}$ isotopes in a number of molecules proved small significant differences to exist; however, they are not larger than one part in ten thousand. This shows that differences in zero-point vibrations etc., for the different isotopes do not affect the field gradient seriously. Isotopic pair ratios Q^1/Q^2 are listed in Table II.

Interpretation of the observed quadrupole couplings is easiest when they represent true

TABLE I. Nuclei investigated with NQR method.

Nucleus	I	Molecule	$eQ_{q_{zz}}$ (Mc/sec)	# (percent)	References
B^{10}	3	$B^{10}(CH_3)_3$	10.36	0	b
B^{11}	3/2	$B^{10}(C_2H_5)_3$	10.43	0.9	
N^{14}	1	ICN^{14}	3.40	0	c
		$N^{14}(CH_3)_6$	4.54	0	
S^{33}	3/2	S^{33}_8	45.8	—	d
Cl^{35}	3/2	Cl^{35}_2	109.0	—	e
Cl^{37}	3/2	CCl^{35}_4	81.9	—	
Cu^{63}	3/2	Cu^{63}_2O	53.4	—	f
Cu^{65}	3/2	$K[Cu^{65}(CN)_2]$	65.3	—	
Ga^{69}	3/2	$Ga^{69}Cl_3$	~58	—	
Ga^{71}	3/2			—	
As^{75}	3/2	$As^{75}_4O_6$	232.5	—	g
Br^{79}	3/2	Br^{79}_2	765	—	h
Br^{81}	3/2	$Br^{79}CH_3$	529	—	i
Sb^{121}	5/2	$Sb^{123}Cl_3$	489	19	j
Sb^{123}	7/2			—	
I^{127}	5/2	I^{127}_2	2153	15	k, l
I^{129}	7/2	SnI^{127}_4	1363	0; 0.9	
Hg^{201}	3/2	$Hg^{201}Cl_2$	724	—	m
Bi^{209}	9/2	$Bi(C_6H_5)_3$	669	8.9	n

* $eQ_{q_{zz}}$ values are given for one isotope of a species only. Corresponding values for the other isotopes may be obtained from the Q ratios given in Table II.

^b H. G. Dehmelt, Z. Physik 133, 528 (1952); 134 (1953).

^c G. D. Watkins and R. V. Pound, Phys. Rev. 85, 1062 (1952).

^d H. G. Dehmelt, Phys. Rev. 91, 313 (1953).

^e R. Livingston, J. Chem. Phys. 19, 803 (1951).

^f H. Krüger and U. Meyer-Berkhout, Z. Physik 132, 171 (1952).

^g H. Krüger and U. Meyer-Berkhout, Z. Physik 132, 221 (1952); H. G. Dehmelt, Phys. Rev. 92, 1240 (1953).

^h H. G. Dehmelt, Z. Physik 130, 480 (1951).

ⁱ H. G. Dehmelt and H. Krüger, Z. Physik 129, 401 (1951).

^j H. G. Dehmelt and H. Krüger, Z. Physik 130, 385 (1951).

^k H. G. Dehmelt, Z. Physik 130, 356 (1951).

^l R. Livingston and H. Zeldes, Phys. Rev. 90, 609 (1953).

^m Dehmelt, Robinson, and Gordy, Phys. Rev. 93, 480 (1954).

ⁿ Robinson, Dehmelt, and Gordy, Phys. Rev. 89, 1305 (1953).

²⁵ The correlation of accurately measured ratios of nuclear electric quadrupole and magnetic dipole moments of isotopes with the same nuclear spin is of value in testing nuclear models. Compare H. Kopfermann, Naturwiss. 38, 29 (1951); A. Bohr and B. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. (1953).

²⁶ Wang, Townes, Schawlow, and Holden, Phys. Rev. 86, 809 (1952).

molecular quantities. Comparison of quadrupole couplings in the gaseous and the solid phase shows that for suitable molecules the influence of the state of aggregation is negligible. It appears that for molecules with small or no dipole moments and small polarizabilities this assumption is quite generally permissible.²⁷ Raman (vibrational) spectra of this class of molecules also are very insensitive to changes in the state of aggregation.

Using Pauling's molecular structure concepts,²⁸ Townes and Dailey²⁹ were able to relate the nuclear quadrupole couplings observed for the same nucleus in a series of molecules to that in the free atom. With the help of this theory, the measured coupling constants can be made to furnish information on the electronic structure of the molecules. Also, in some cases, where it is difficult to obtain atomic couplings experimentally, good approximate values can be deduced from couplings in suitable molecules (compare Table III). The problem of evaluat-

TABLE II. Quadrupole moment ratios of isotopic pairs.

Isotope		Q_1/Q_2			
1	2	NQR	Reference	Other	Source
B ¹⁰	B ¹¹	2.084 ±0.002	a	2.17	g
Cl ³⁵	Cl ³⁷	1.26878 ±0.00015	b	1.2686 ±0.0004	h
Cu ⁶³	Cu ⁶⁵	1.0806 ±0.0003	c	1.08 ±0.02	i
Br ⁷⁹	Br ⁸¹	1.1970 ±0.0002	d	1.19707 ±0.00003	j
Sb ¹²¹	Sb ¹²³	1.2751 ±0.0002	e	1.263 ±0.01	k
I ¹²⁹	I ¹²⁷	0.701213 ±0.000015	f	(0.7353)	l

^a H. G. Dehmelt, Z. Physik **133**, 528 (1952); **134** (1953).
^b R. Livingston, Phys. Rev. **82**, 289 (1951).
^c H. Kruger and U. Meyer-Berkhout, Z. Physik **132**, 171 (1952).
^d H. G. Dehmelt and H. Krüger, Z. Physik **129**, 401 (1951).
^e H. G. Dehmelt and H. Krüger, Z. Physik **130**, 385 (1951).
^f R. Livingston and H. Zeldes, Phys. Rev. **90**, 609 (1953).
^g Gordy, Ring, and Burg, Phys. Rev. **78**, 512 (1950) (hfs of rotational lines).
^h V. Jaccarino and J. G. King, Phys. Rev. **83**, 471 (1951) (atomic beam experiment).

ⁱ Bleaney, Bowers, and Ingram, Proc. Phys. Soc. (London) **A64**, 758 (1951) (paramagnetic resonance hfs).

^j J. G. King and V. Jaccarino, Bull. Am. Phys. Soc. **28**, No. 2, 11 (1953) (atomic beam experiment).

^k C. C. Loomis and M. W. P. Strandberg, Phys. Rev. **81**, 798 (1951) (hfs of rotational lines).

^l Livingston, Gilliam, and Gordy, Phys. Rev. **76**, 149 (1949) (hfs of rotational lines).

²⁷ Compare C. A. Coulson, *Valence* (Oxford, Clarendon Press, 1952), Ch. 10.2; G. Herzberg, *Infrared and Raman Spectra* (D. Van Nostrand Company Inc., New York, 1945), Chapter V, 2.

²⁸ Linus Pauling, *Nature of the Chemical Bond* (Oxford University Press, London, 1940).

²⁹ C. H. Townes and B. P. Dailey, J. Chem. Phys. **17**, 782 (1949).

TABLE III.

Nu- cleus	Elec- tronic con- figuration	$ eQ_{zz}(\text{atom})/\hbar $ ^a		$ Q ^b$	Q^b	References	
		From NQR	From other experiments			NQR	Other
B ¹⁰	$2s^22p$	10.31	7.5	0.111	+0.06		
B ¹¹		4.95	3.4	0.053	+0.03	e	j
S ³³	$3s^23p^1$	50.4	57 ... 109	0.053	-0.055 ... 0.103	d	k
S ³³		35.0	40 ... 76	0.037	+0.038 ... 0.072		
Cl ³⁵	$3s^23p^3$	109.0	109.746	0.076	-0.07894		
Cl ³⁷		86.0	88.510	0.060	-0.06213	e	l
Br ⁷⁹	$4s^24p^5$	765	789.756	0.303	+0.335	f	m
Br ⁸¹		639	643.032	0.28	+0.290		
I ¹²⁷	$5s^25p^5$	2280	2292.430	0.75	-0.75	g	n
I ¹²⁹		1600	1607.481	0.53	-0.53	h	
Hg ²⁰¹	$6s6p$	1000	780 (from 3P_2 term)	0.6	-0.2 ... 0.5	i	o

^a Per unbalanced p electron in magnetic substate $m_l=0$.

^b Relativistic corrections as well as the Sternheimer correction—Phys. Rev. **80**, 316 (1952)—have been applied when appreciable.

^c H. G. Dehmelt, Z. Physik **133**, 528 (1952); **134** (1953).

^d H. G. Dehmelt, Phys. Rev. **91**, 313 (1953).

^e R. Livingston, Phys. Rev. **82**, 289 (1951).

^f H. G. Dehmelt, Z. Physik **130**, 480 (1951).

^g H. G. Robinson and W. Gordy, J. Chem. Phys. (to be published).

^h R. Livingston and H. Zeldes, Phys. Rev. **90**, 609 (1953).

ⁱ Dehmelt, Robinson, and Gordy, Phys. Rev. **93**, 480 (1954).

^j Gordy, Ring, and Burg, Phys. Rev. **78**, 512 (1950) (hfs of rotational lines).

^k C. H. Townes and B. P. Dailey, J. Chem. Phys. **17**, 782 (1949) (hfs of rotational lines).

^l V. Jaccarino and J. G. King, Phys. Rev. **83**, 471 (1951) (atomic beam experiment).

^m J. G. King and V. Jaccarino, Bull. Am. Phys. Soc. **28**, No. 2, 11 (1953) (atomic beam experiment).

ⁿ Measurements by King, Stroke, and Jaccarino. The author is indebted to Dr. Jaccarino for private communication of this result prior to publication.

^o Schuler and Schmidt, Z. Physik **98**, 239 (1935); K. Murakawa and S. Suwa, J. Phys. Soc. Japan **5**, 429 (1950) (optical spectra hfs).

ing a nuclear quadrupole moment eQ can thus first be reduced to the atomic case, where established procedures^{29a} to obtain q_{zz} are available. The expression for the field gradient q_{zz} produced by a single atomic electron outside a closed shell

$$q_{zz} = e \int \psi^*(3 \cos^2\theta - 1) r^{-3} \psi dV \quad (F1)$$

can be split in two factors, namely, the easily calculated average of $3 \cos^2\theta - 1$ and the more difficult average of r^{-3} over the electron charge distribution. Here r represents the distance nucleus-electron and θ is the azimuthal angle against z , the axis of quantization. One can now try to evaluate $\langle r^{-3} \rangle_N$ from atomic Hartree functions or, simpler, make use of the fact that it can be calculated from the experimentally accessible optical fine structure doublet separation $\Delta\nu$. The

^{29a} Compare, e.g., H. Kopfermann, *Kernmomente* (Akademische Verlagsgesellschaft, Leipzig, 1940), Sec. 13.

latter procedure leads to:

$$(q_{zz})_{l,m_l} = - (4e/R\alpha^2 a_0^2) (\Delta\nu/Z_i) \cdot [3m_i^2 - l(l+1)]/(2l-1)(2l+1)(2l+3). \quad (F2)$$

Here l, m_l are the quantum numbers of the electron orbital angular momentum and its z component. Z_i is an effective nuclear charge which is approximately equal to $Z-4$ for p and $Z-11$ for d electrons.

In this way it is possible to obtain moderately accurate absolute values of nuclear quadrupole moments from NQR spectra; signs however cannot be determined. In cases where perturba-

tions due to neighboring molecules cannot be neglected, e.g., for solid I_2^{30} —indicated by the large asymmetry parameter ϵ —it is still possible to apply more refined procedures.³¹ The quadrupole moments obtained up to now from NQR are given in Table III.

The author wishes to express his thanks to Mr. H. G. Robinson and Mr. T. C. Chen for suggestions as to improvements of the manuscript.

³⁰ H. G. Dehmelt, Z. Physik **130**, 356 (1951); R. V. Pound, Am. Phys. Soc. Bull. **26**, 56 (1951).

³¹ C. H. Townes and B. P. Dailey, J. Chem. Phys. **20**, 35 (1952).

Electromagnetism without Magnetism: An Historical Sketch

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(Received July 21, 1953)

Maxwell's formulation of electromagnetism is not the only formulation, though it is the most familiar. In particular, the whole subject of electrodynamics can be expressed as an extension of Coulomb's equation, with the magnetic field entirely eliminated. The paper reviews the history of this attack, beginning with the work of Coulomb, Ampère, and Gauss, and concluding with recent developments.

1. INTRODUCTION

ONE of the most familiar parts of physics deals with the magnetic field and with the visualization of flux lines, tubes, and magnetic equipotentials. Without question, Faraday's visualization of magnetic phenomena has been a fruitful one in engineering practice, yet the whole concept of a magnetic field is a fiction. The magnetic-field fiction is perhaps most helpful in the design of heavy electric machinery and least helpful in the more modern applications dealing with moving particles. What is the magnetic field about a moving electron, for instance, and why does that field disappear when the observer moves with the electron?

It is interesting to note, therefore, that we are not necessarily limited to the customary ideas of electromagnetism but may formulate an electrodynamics that is entirely free from the magnetic-field concept. The observed quantities are *forces* (or emf's, which are integrated forces per unit charge)—not magnetic flux lines or flux densities.

The pre-Maxwellian approach is on this phenomenological plane. It deals with *forces on particles*, starting with Coulomb's equation and adding terms to include velocity and acceleration of the charges.

Like so many ideas in mathematical physics, this conception of a single basic equation for electrodynamics seems to have originated in the fertile brain of Karl Friedrich Gauss. He worked on the subject in 1835, but the first published equation of this type was presented by Weber in 1841.

Consider two charges, Q_1 and Q_2 , separated at a given instant by the distance r . Evidently, a complete formulation of electrodynamics is obtained by expressing the force under the following conditions:

- (a) No relative motion
- (b) Uniform relative motion } constant charge
- (c) Accelerated motion
- (d) Variable charge.

The purpose of the paper is to outline the development of a single equation that includes these four cases. The equation is equivalent in many respects to the four equations of Maxwell, but it replaces the fictitious **E**, **D**, **B**, and **H** by the measurable quantity—a force. In some applications, the single equation may be found more direct and unambiguous than Maxwell's equations; in other applications, the customary procedures may be preferable. But in any case, the physicist and the engineer should know that such a phenomenological approach exists and has been worked out by Coulomb, Ampère, Gauss, Weber, Riemann, and Ritz.

The formulation outlined in this paper is relativistic in the Galilean sense: "absolute velocity" is considered meaningless, all velocities entering the equations are *relative* velocities. Other similar but nonrelativistic studies by Clausius, Stefan, F. E. Neumann, C. Neumann, and Helmholtz lean heavily on the idea of an aether and are ignored in the present paper. The question of high-velocity particles is also omitted. All equations are expressed in the rationalized mks system.

2. PRE-MAXWELLIAN WORK

The first step in the establishment of the basic equation of electrodynamics was taken in 1767, when Priestley¹ inferred that the force between charged particles varied inversely as the square of the distance between them. A crude experimental proof was obtained by Robison² in 1769, and a more exact verification was found by means of the torsion balance of Coulomb³ in 1785.

Coulomb's equation is

$$\frac{\mathbf{F}}{Q_2} = \mathbf{a}_r \frac{Q_1}{4\pi\epsilon r^2}, \quad (1)$$

where **F** is the force (newton) on Q_2 , \mathbf{a}_r is a unit vector pointing from Q_1 to Q_2 , and ϵ is the permittivity (farad m^{-1}). Charge Q_1 is stationary

¹ Joseph Priestley, *The History and Present State of Electricity, with Original Experiments* (London, 1767), p. 732.
² John Robison, *Mechanical Philosophy* (Edinburgh, 1769), Vol. IV, p. 73.

³ C. A. Coulomb, "Ou l'on détermine suivant quelle loi le fluide magnétique ainsi que le fluide électrique agissent soit par répulsion, soit par attraction," *Mém. acad.* (1785); *Collection de mém.* (Paris, 1884), I, p. 116.

with respect to Q_2 , though both charges may be in motion with respect to the laboratory. Evidently Eq. (1) is the first step toward a general equation of electrodynamics.

The next step consists in adding a term to represent the additional force caused by relative motion of the charges. In Maxwell's theory, this additional force is supposed to be associated with the magnetic field. But Ampère represented the experimental results without postulating a field.

Ampère⁴ performed four experiments on the forces exerted by current-carrying conductors. On the basis of these experiments, he deduced an equation for the force between *current elements*:

$$d^2\mathbf{F} = \mathbf{a}_r \frac{|I_1| |I_2| d\mathbf{s}_1 d\mathbf{s}_2}{4\pi\epsilon c^2 r^2} \times [2 \sin\theta_1 \sin\theta_2 \cos\eta - \cos\theta_1 \cos\theta_2], \quad (2)$$

where I_1 and I_2 are currents in elementary conductors $d\mathbf{s}_1$ and $d\mathbf{s}_2$. The distance between elements is r , the angle between $d\mathbf{s}_1$ and \mathbf{a}_r is called θ_1 , and the angle between $d\mathbf{s}_2$ and \mathbf{a}_r is θ_2 . The vectors \mathbf{a}_r and $d\mathbf{s}_1$ determine a plane, which makes an angle η with the plane determined by \mathbf{a}_r and $d\mathbf{s}_2$. Ampère proved that Eq. (2) is the only possible equation that satisfies the four experiments and, in addition, the postulate that the force acts along the line of centers.

Ampère's equation is not exactly what we want; for it expresses a force between current elements, not a force between moving charges. A direct current, however, is merely a set of charges drifting at uniform velocity; so Eq. (2) is easily translated into an equation for force between charges. This step was taken by Gauss in 1835, though the material was not published⁵ until 1867. Gauss' equation is

$$\frac{\mathbf{F}}{Q_2} = \mathbf{a}_r \frac{Q_1}{4\pi\epsilon r^2} \left[1 + \left(\frac{v}{c} \right)^2 (1 - \frac{3}{2} \cos^2\theta) \right], \quad (3)$$

where v is the magnitude of the relative velocity of Q_1 with respect to Q_2 , and θ is the angle between \mathbf{v} and \mathbf{a}_r (Fig. 1). The first term of Eq. (3) represents the *Coulomb force*. The remainder of the equation represents the *Ampère force*, which depends on the relative velocity of the particles.

⁴ A.-M. Ampère, *Mém. acad. sci.* VI, 175–388 (1823).
⁵ C. F. Gauss, "Zur mathematischen Theorie der elektrodynamischen Wirkung," *Werke* (Göttingen, 1867), Vol. V, p. 602.

Although Gauss did not seem to be satisfied with his equation, he undoubtedly discussed the subject with his students and colleagues. Both Weber and Riemann were introduced to the problem by Gauss, and both made contributions. In 1846, Weber⁶ developed an equation of electrodynamics, including terms for both velocity and acceleration. Instead of Gauss' formula for the Ampère force, Weber advocated

$$\mathbf{a}_r \frac{Q_1}{4\pi\epsilon r^2} \frac{1}{2} (v/c)^2 \cos^2\theta.$$

In his lectures at Göttingen in 1861, Riemann suggested a different equation with a velocity term in the direction of the relative velocity and an acceleration term in the direction of the acceleration. This contribution was published⁷ in 1876. The velocity terms are

$$\frac{Q_1(v/c)^2}{4\pi\epsilon r^2} [\mathbf{a}_r - \mathbf{a}_r 2 \cos\theta],$$

where \mathbf{a}_r is a unit vector in the direction of \mathbf{v} .

Both the Weber and Gauss expressions give correct results for closed circuits if one assumes with Fechner⁸ that a current consists of equal quantities of positive and negative electricity moving in opposite directions at the same velocity. In the middle of the 19th century, this seemed a reasonable assumption; but modern ideas on metallic conduction show that only the Gauss equation is tenable. A recent study⁹ proves that the only equation that agrees with

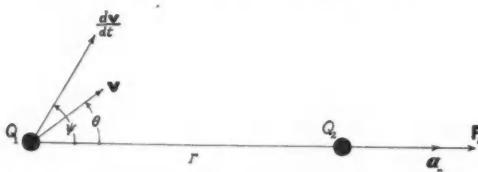


FIG. 1. Two charged particles separated by distance r . The force \mathbf{F}_2 on charge Q_2 depends on the magnitudes of Q_1 and Q_2 and also on the relative velocity \mathbf{v} of Q_1 with respect to Q_2 and on the acceleration $d\mathbf{v}/dt$.

⁶ Wilhelm Weber, "Elektrodynamische Maassbestimmungen über ein allgemeines Grundgesetz der elektrischen Wirkung," *Werke* (Julius Springer, Berlin, 1893), Vol. 3, p. 25.

⁷ B. Riemann, *Schwere, Elektrizität, und Magnetismus* (C. Rümpler, Hannover, 1876), p. 327.

⁸ G. T. Fechner, *Ann. Physik* **64**, 337 (1845).

⁹ Moon and Spencer, "The Coulomb force and the Ampère force," *J. Franklin Inst.* **255** (1953).

both the Ampère experiments and modern electronics is Eq. (3). This equation was independently derived¹⁰ by Bush in 1926.

The Gauss equation may be employed to determine the force between charged particles, whether these particles be stationary or in uniform relative motion. The equation also gives the force between conductors carrying direct current, also induced emf's produced by uniform motion of one current-carrying conductor with respect to another.⁹ But with alternating currents, we have *accelerated* charges and another term must be added to the equation.

Weber's equation,⁶

$$\frac{\mathbf{F}}{Q_2} = \mathbf{a}_r \frac{Q_1}{4\pi\epsilon r^2} \left[1 - \frac{1}{2} \left(\frac{v}{c} \right)^2 \cos^2\theta + \frac{r}{c^2} \left(\frac{dv}{dt} \right) \cos\psi \right], \quad (4)$$

is remarkable in that it is the first electrodynamic equation to contain an acceleration term. (The quantities θ and ψ are defined in Fig. 1.) Weber's expression gives the correct induced emf and allows the calculation of inductance. The fact that it fails when applied to open-circuit problems could not have been foreseen in Weber's day, when radiation from antennae had not been dreamed of. The equation was formulated on the basis of energy conservation and must necessarily fail when there is radiation.

The complete equation of Riemann⁷ is

$$\frac{\mathbf{F}}{Q_2} = \frac{Q_1}{4\pi\epsilon r^2} \left\{ \mathbf{a}_r \left[1 + \left(\frac{v}{c} \right)^2 \right] - \mathbf{a}_r 2 \left(\frac{v}{c} \right)^2 \cos\theta + \mathbf{a}_a \frac{2r}{c^2} \left(\frac{dv}{dt} \right) \right\}. \quad (5)$$

It is also unsatisfactory.

3. RETARDATION

Riemann was the first to make a definite statement that "action is not instantaneous but is propagated at a constant velocity c ." In a paper read¹¹ in 1858 but not published until 1867, he replaces $Q(t)$ in the Coulomb force by $Q(t-r/c)$ and gives a definite formulation of retardation. Perhaps Riemann obtained the germ

¹⁰ V. Bush, *J. Math. Phys.* **5**, 129 (1926).

¹¹ B. Riemann, *Phil. Mag.* **34**, 368 (1867); *Gesammelte mathematischen Werke* (B. G. Teubner, Leipzig, 1892), p. 288.

of his idea from Gauss, but to Riemann certainly belongs the credit for the first precise statement.

The idea of retardation is generally credited to Ludwig Lorenz who introduced the retarded potentials¹² in 1867. Evidently when charge is a function of time, the force produced at a distant point should be obtained by introducing retardation into the electrodynamic equation. For short distances, however, r/c may be negligible. Thus for much of electrical engineering, the electrodynamic equation applies directly; and it is only at very high frequencies or very large distances that the further refinement must be introduced of replacing $Q_1(t)$ by $Q_1(t-r/c)$.

4. POST-MAXWELLIAN WORK

Gauss, Riemann, and Weber realized that their formulation of electrodynamics was incomplete. Indeed, the investigation could not be pushed further without the theory of electronic conduction and the discovery of electromagnetic radiation. Maxwell¹³ was well-acquainted with the work done on the continent, and he devoted a chapter in his book to this subject. He chose, however, to break away from the Continental approach and to base his theory on the electric and magnetic fields of Faraday. Maxwell's theory did not meet universal approbation. Lord Kelvin never accepted it, the Germans continued for some time to follow Weber, and Poincaré said,¹⁴

"La première fois qu'un lecteur français ouvre le livre de Maxwell, un sentiment de malaise, et souvent même de défiance se mêle d'abord à son admiration."

But, largely because of the experiments of Hertz,¹⁵ the Maxwellian theory triumphed and the electric and magnetic fields became cornerstones of physics. Lorentz¹⁶ attempted to reconcile the field ideas with the older interparticle force concept (which seemed to be almost necessary in view of the growing knowledge of electrons), but the Lorentz theory was not successful.

¹² L. Lorenz, Phil. Mag. **34**, 287 (1867); Ann. Physik **131**, 243 (1867).

¹³ J. C. Maxwell, *Treatise on Electricity and Magnetism* (Oxford University Press, London, 1904), Chap. XXIII.

¹⁴ H. Poincaré, *Électricité et Optique* (Carre et Naud, Paris, 1901), p. III.

¹⁵ H. Hertz, *Electric Waves* (Macmillan and Company, Ltd., London, 1893).

¹⁶ H. A. Lorentz, *Theory of Electrons* (Leipzig, 1909).

The outstanding attempt to return to the Weber ideas was made by Ritz¹⁷ in 1908. His equation is

$$\frac{\mathbf{F}}{Q_2} = \frac{Q_1}{4\pi\epsilon r^2} \left\{ \mathbf{a}_r \left[1 + \frac{3-k}{4} \left(\frac{v}{c} \right)^2 \right. \right. \\ \left. \left. - \frac{3(1-k)}{4} \left(\frac{v}{c} \right)^2 \cos^2\theta \right] - \mathbf{a}_v \frac{k+1}{2} \left(\frac{v}{c} \right)^2 \cos\theta \right. \\ \left. - \mathbf{a}_r \frac{r}{2c^2} \left(\frac{dv}{dt} \right) \cos\psi - \mathbf{a}_a \frac{r}{c^2} \left(\frac{dv}{dt} \right) \right\}. \quad (6)$$

The first term represents the Coulomb force; the second, third, and fourth terms represent the Ampère force. The arbitrary constant k was to be determined eventually by experiment. But since the Ampère force must agree⁹ with the Gauss equation, the only possible value of k is -1 , and Eq. (6) becomes

$$\frac{\mathbf{F}}{Q_2} = \frac{Q_1}{4\pi\epsilon r^2} \left\{ \mathbf{a}_r \left[1 + \left(\frac{v}{c} \right)^2 \left[1 - \frac{3}{2} \cos^2\theta \right] \right. \right. \\ \left. \left. - \mathbf{a}_r \frac{r}{2c^2} \left(\frac{dv}{dt} \right) \cos\psi - \mathbf{a}_a \frac{r}{c^2} \left(\frac{dv}{dt} \right) \right] \right\}. \quad (6a)$$

Recently (1946) a similar formulation has been made by Warburton,¹⁸ who finds

$$\frac{\mathbf{F}}{Q_2} = \frac{Q_1}{4\pi\epsilon r^2} \left\{ \mathbf{a}_r \left[1 + \left(\frac{v}{c} \right)^2 \right. \right. \\ \left. \left. \times [(1-A) - \frac{3}{2}(1-2A) \cos^2\theta] \right] - 2\mathbf{a}_v A \left(\frac{v}{c} \right)^2 \cos\theta \right. \\ \left. + \mathbf{a}_r (1-2A) \frac{r}{c^2} \left(\frac{dv}{dt} \right) \cos\psi - \mathbf{a}_a \frac{2Ar}{c^2} \left(\frac{dv}{dt} \right) \right\}. \quad (7)$$

Setting $A=0$, which is the only way of obtaining the correct Ampère force,⁹ we obtain

$$\frac{\mathbf{F}}{Q_2} = \frac{Q_1}{4\pi\epsilon r^2} \left\{ 1 + \left(\frac{v}{c} \right)^2 \left[1 - \frac{3}{2} \cos^2\theta \right] \right. \\ \left. + \frac{r}{c^2} \left(\frac{dv}{dt} \right) \cos\psi \right\}. \quad (7a)$$

¹⁷ Walther Ritz, Ann. chim. et phys. **13**, 145 (1908); *Gesammelte Werke* (Paris, 1911), p. 317; W. Hovgaard, J. Math. Phys. **11**, 218 (1932).

¹⁸ F. W. Warburton, "Reciprocal electric force," Phys. Rev. **69**, 40 (1946).

Both Eq. (6a) and Eq. (7a) give the correct Coulomb force and the correct Ampère force. Thus these equations can be used to predict forces on charges moving at uniform relative velocity, forces between current elements, forces between complete circuits carrying direct currents, and induced emf's. Since neither equation contains anything corresponding to Maxwell's displacement current, however, we can hardly expect Eqs. (6a) and (7a) to be of universal validity. Indeed, application of these equations to a simple antenna problem leads to incorrect results.¹⁹

Evidently if the equation of electrodynamics is to cover the variety of problems handled by Maxwell's equations, it must contain a term applying to charge that is a function of position and time. Such a modified equation¹⁹ is

$$\begin{aligned} \frac{\mathbf{F}}{Q_2} = & \mathbf{a}_r \frac{Q_1}{4\pi\epsilon r^2} \left(\frac{v}{c} \right)^2 [1 - \frac{3}{2} \cos^2\theta] \\ & - \mathbf{a}_a \frac{Q_1}{4\pi\epsilon c^2 r} \frac{d}{dt} v(t-r/c) \\ & - \mathbf{a}_r \frac{1}{4\pi\epsilon} \frac{\partial}{\partial r} \left[\frac{1}{r} Q_1(t-r/c) \right]. \quad (8) \end{aligned}$$

¹⁹ Moon and Spencer, "A new electrodynamics," J. Franklin Inst. (to be published).

The first term is the familiar Gauss equation (3). The Coulomb equation (1) is contained in the last term. But if Q_1 is a function of time, as it will be in an antenna, for instance, the last term adds important additional information. Equation (8) gives results,¹⁹ for the loop antenna and the dipole antenna, that are identical with those obtained by using Maxwell's equations.

5. CONCLUSIONS

The electrodynamic equation, based on interparticle forces, has been in the process of development for almost two centuries. The latest development, Eq. (8), appears to subsume all of electrodynamics and to have in some cases advantages over Maxwell's equations:

- (1) It provides a direct approach to particle dynamics, free from much of the mathematical subtlety associated with a field theory.
- (2) It eliminates completely the concept of a magnetic field.
- (3) It is relativistic in the Galilean sense. Maxwell's equations are based on an aether and on *absolute* velocities. To make them relativistic, one must modify the basic concepts of space and time, as was done by Einstein in 1905. With the equations of this paper, only *relative* velocities appear and no Einstein revolution is necessary.

Research Corporation Grants

Research Corporation is a nonprofit foundation which supports scientific research through grants-in-aid to colleges, universities and scientific institutions. It was established in 1912 by the late Frederick Gardner Cottrell, scientist and inventor, with the gift of his patent rights in the field of electrical precipitation.

In its early years the foundation derived its income almost entirely from licensing the Cottrell patents to industry. Later it began to manufacture and sell precipitation equipment. The chief uses of the equipment which removes materials suspended in gases are for reduction of air pollution, recovery of valuable products, or treatment of gases for further industrial use. This manufacturing

activity continues today as a major source of income for the grants programs of the foundation.

Since 1912 Research Corporation has distributed nearly \$8,000,000 in the form of grants-in-aid of research, generally in relatively small sums of money advanced at early, critical stages of researches when other support could not be attracted. The grants have played a significant part well beyond their dollar value in specific accomplishment, in the general advancement of scientific knowledge and in the training of young scientists.

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Physics in Physiology*

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(Received September 15, 1953)

The importance of physics to the physician is discussed. It is pointed out that the routine process of physical examination of a patient is, in essence, based on physical principles. A few applications of physics made during every medical student's education are interpreted from the point of view of man as a machine.

I SHOULD like to direct your attention to the physician and to his precursor, the medical student, in relation to the subject of physics. As physics teachers, you know that your subject lacks strong appeal to premedical students generally. As teachers in medical school, my colleagues and I know that too many of our students, despite what may be assumed as adequate instruction, fail to recognize simple physical problems and fail to solve them without extra assistance. As a concrete, if somewhat extreme, example of the latter situation, I can relate in all seriousness that out of every hundred medical students whom I have served in the last two decades, two or three have been unable on first trial to connect dry cells in series even with the aid of a diagram, while to a considerably larger number simple applications of the gas laws have presented rather a major challenge. Since medical students are not drawn from the lower segments of college classes, and since a year of college physics is a uniform admissions requirement, one may be legitimately concerned over the mediocre grasp of physical fundamentals which frequently results.

There is little reason to doubt that we are dealing here with a problem in motivation. Intelligent people are exposed to a subject of moderate difficulty, learn enough to satisfy their instructors, and then dismiss the entire matter as an unpleasant and inconsequential hurdle successfully cleared. Unfortunately, defining the problem does not solve it, and I am not so naïve as to suggest that stimulation of the will to learn is a simple task. However, it has seemed to me that the traditional concept of physics as a science dealing largely with inanimate materials

is singularly uninspiring to the person who, lacking concrete knowledge of the academic content of medical studies, thinks of his future work largely in biologic terms. If the premedical student had any convincing reason to visualize the practice of medicine in terms of physics, surely his resistance to learning the subject with an intent to preserve his learning would be lowered appreciably.

Actually, it is easy to make a good case for physics in medicine from the daily activities of the physician. He is usually not long at the sick-bed before he pops a thermometer into the patient's mouth and improves the moments of waiting by feeling the pulse. He then proceeds merrily with divers peerings, proddings, thumpings, and soundings which constitute a system he learned in medical school under the name of *physical diagnosis*, or, more correctly, *physical examination*.

The art of physical examination is, as the name accurately implies, the study of the patient by physical means, with the purpose of observing physical phenomena. It is the physician's primary approach to diagnosis, and many hours are devoted to acquiring the requisite skill in this important branch of medical physics. Formally, physical examination combines four physical processes. The first is called inspection and consists simply in visual observation. The next, called palpation, is manual exploration—feeling with the hands. The third process is called auscultation, i.e., listening, usually through a simple instrument quite erroneously named the stethoscope. The final process is percussion, which is the production of sound by tapping over the surface of the body. Special instruments have been designed to refine and extend these maneuvers, for example, the apparatus of roentgenog-

* Presented at the 1953 Summer meeting of AAPT, Pittsburgh, Pennsylvania, June 25.

raphy and electrocardiography, but the physical character remains clear.

Figure 1 shows a collection which I like to think of as the basic instrumentation for the physician. These are all physical instruments, and they involve the subjects of light, heat, mechanics, hydraulics, and sound. They are employed almost routinely to supplement the physician's hands, eyes, and ears in the examination of patients, and their skillful use, backed by the requisite brain power and medical knowledge, will help to reveal the nature of physical breakdown in many instances.

It must be obvious that, while the processes just described are extremely simple, the physics which they are designed to study may be most complex. In place of more dramatic and sophisticated examples of physics as applied in medicine, I desire to emphasize that the physician is a physicist daily and unavoidably.

Turning now to the medical student, we find perhaps the most extensive and diversified applications of physics in the subject matter of physiology. Physiology may be defined broadly as the study of function, and as such it underlies the entire structure of medicine. Health is actually an integral of normal function, and

disease, in one phase or another, must be looked upon as aberrant function. The scientific content of physiology is a curious mixture of all of the natural sciences and leans with particular weight upon anatomy, physics, and chemistry, not necessarily in the order named. Physiology attempts to answer the usual questions: "How?" and "How much?" It will cheerfully pirate the concepts and techniques of any scientific discipline in order to achieve its goals and is neither too proud to use the humblest tools nor frightened by the most elaborate.

The relationship of physics to the concepts of physiology is readily appreciated by recalling that the body is really an aggregation of levers and assorted plumbing—a machine in the strictest sense. In organization, the body resembles a factory more than the steam or internal combustion engine with which it is so frequently compared. It is not only a single machine but a system of machines, the latter rather accurately oriented from a functional standpoint, supplied with fuel by a continuous, fluid distribution network and controlled by a communications system unrivaled among the works of man. Further, these various machines are designed to carry out the necessary energy transformations at or very close to the output locus, so that the transmission problems, while not eliminated, are at least localized. The very popular feed-back principle for controlling levels of operation is an integral part of the total mechanism, furnished at no extra cost. The idea of the body as a machine is therefore valid and well established and places function squarely in the domain of classical physics.

It has been aptly said that man can do nothing but move things. Certainly, the useful resultant of all physiologic activity is some sort of motion. The sum total of overt movement we call behavior, and this is of great interest to historians, sociologists, and economists as well as to physicians. Now, in the body, the sole source of movement above the molecular level is muscle, a tissue which has the property of mechanical shortening and which, not incidentally, makes up more than one-half of the body mass. The most familiar evidence of muscular activity is of course the motion of limbs, jaws, and tongue, but it may be fairly stated that every vital

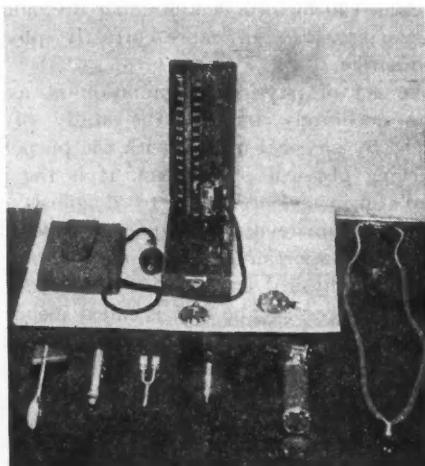


FIG. 1. Basic instrumentation for the physician: aids in physical examination. On white background: sphygmomanometer, watch, ophthalmoscope. On dark background: percussion hammer, flashlight, tuning fork, thermometer, otoscope (for inspecting the external ear canal and eardrum membrane), stethoscope.

activity is supported by rather gross and obvious kinetics. Air must be pumped into and out of the lungs, blood must be circulated to every tissue in adequate amounts, food and the products of digestion must be moved along the alimentary tract, and of course the bony levers of the skeleton must be actuated to produce the slightest bodily movement. Hence, it is not surprising that physiologists have long been preoccupied with this contractile tissue and have been at no little pains to describe its activity quantitatively. This type of study is largely physical. Even though, of late, the chemists have with great ingenuity learned some important things about the way in which



FIG. 3. Top curve, record of contraction of an isolated muscle in response to a single electrical stimulus. Wavy line is record of a tuning fork at 100 c/sec. Vertical line *s* marks the point of stimulation. Two lower graphs, composition of tetanus. Record of contraction of isolated muscle in response to electrical stimulation at frequencies of 1 to 60 per sec. Along the middle graph, 30, 40, 50, 60 c/sec; along the bottom graph, 1, 10, 15, 20, 25 c/sec. Duration of stimulus burst is 2 sec in each case. See text.

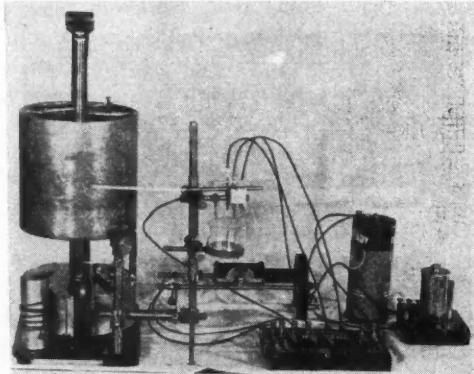


FIG. 2. Typical student assemblage for study of muscular contraction. Shortening of the muscle elevates the horizontal lever, which records the contraction on smoked paper (not shown) covering the kymograph drum. Electrical stimuli are supplied by an induction coil. Stimulus strength is closely regulated by a potentiometer (extreme right) in parallel with the primary of the coil.

muscle may transform chemical energy into mechanical work, they cannot avoid the physical implications of the contraction and, even more baffling, of the relaxation. I submit, then, that a good grounding in mechanics has a wealth of meaning for the physiologist.

As indicated above, no field of physics is without application in physiology, and the multiplication of examples would serve no useful purpose. Simply to fill in the picture, we make mention of the relation between optics and vision, sound and hearing, hydraulics and the circulation of body fluids, diffusion and the transport of ions and molecules across membranes, the behavior

of gases in relation to the mechanics of respiration, and electricity as applied to a wide variety of important physiologic measurements.

Since so much of physiology is physics, it is not surprising that the physiologist's instruments lean strongly toward the physical side. We have already emphasized motion as a sort of common end product of physiologic activity. The measurement of motion is a favorite physiologic method, and as time is an important variable the moving surface has become, as much as any one device, the hall mark of physiologic investigation. Figure 2 shows a typical apparatus assemblage (in fact, that employed by our medical students) for studying the contraction of muscle. The electrical assembly for stimulating the muscle (or motor nerve) is also included. This apparatus, if one may contend with the chemists for the battery, is entirely physical. Examples of records obtained with this sort of equipment are shown in Fig. 3. The top curve in Fig. 3 represents the contraction of an isolated muscle in response to a single electrical stimulus. The vertical line *s* marks the point of stimulation and is followed by a period of apparent inactivity called the graphic latent period. The tissue then shortens and relaxes abruptly, describing a rather typical curve.



FIG. 4. Record of respiratory pressure (upper tracing) and arterial blood pressure (second tracing) in a dog, showing effects of vagal stimulation. See text.

Physiologists are interested in the time relations of this contractile process as well as in the extent of shortening, the work potential, the heat evolved, and many other concomitants of the activity. The two lower curves show how a sustained contraction is developed by fusion of a series of individual contractions. It is clear that the frequency of stimulation required to produce a smooth, sustained shortening is related to the speed of the single twitch. The amplitude of the sustained contraction increases with increasing frequency of stimulation, but there is a limit to such gain, reached in the illustrated case between 40 and 50 stimuli per second. All voluntary muscular contraction is of this sustained or tetanic type, but the mammalian forms have a more economical mechanism of achieving the same result by summation of asynchronous, sub-tetanic contractions of parts of a muscle.

Frequently, one wishes to study the gross effects of various procedures as they apply to the entire organism. For this purpose, the study of the vital functions of respiration and circulation is most revealing. The illustration selected here (Fig. 4) shows simultaneous records of respiratory pressure (upper tracing) and arterial blood pressure (second tracing) during stimulation of a vagus nerve, a procedure which tends to inhibit the heart beat. The effects of even temporary cessation of heart contraction are dramatically portrayed. This is a little study in hydraulics.

For an example from the electrical field, we may turn to the so-called electrocardiograph. Tissue activity, especially nervous and muscular

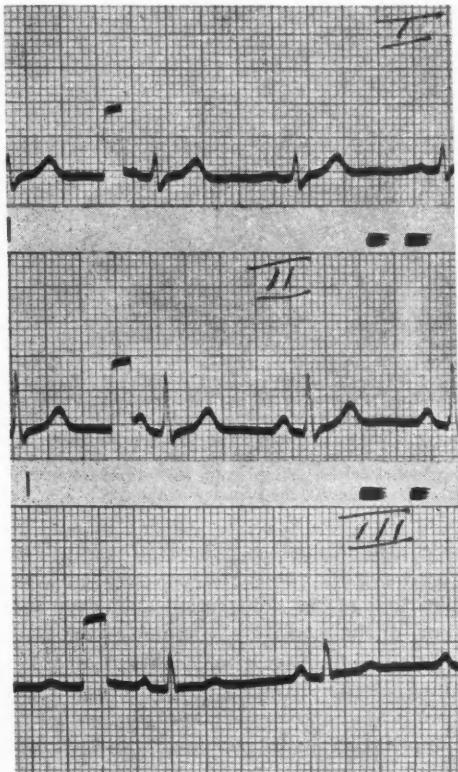


FIG. 5. Normal human electrocardiogram. Each repeated series of deflections represents the resultant action potential for one cycle of cardiac activity. Variations in pattern among the several records are due to altering the position of the galvanometer electrodes on the surface of the body. Time is indicated in 0.2-sec intervals (heavier coordinate lines).

activity, is accompanied by a state of electrical imbalance which can be detected by suitably designed galvanometers even from electrical contacts with the surface of the body. The action potentials of the heart are readily recorded in this manner, as shown in Fig. 5. The first instrument used for this purpose was the capillary electrometer. This was replaced by the string-galvanometer of Ader, popularized by Einthoven, which dominated the field until the relatively recent manufacture of electronic instruments for the purpose. It should perhaps be emphasized that the deflections shown in the record do not represent mechanical contraction of the heart, but rather the spread of an excitatory process through the cardiac musculature. This sort of study is now applied routinely by cardiologists.

Before leaving the heart field, reference to a fairly new instrument may be of interest. The forcible ejection of blood from the heart into the aorta, the latter curving sharply upon itself within a few centimeters of its origin, causes the entire body to oscillate in a head-to-foot direction. If a subject lies on a suitably designed bed or table, such oscillations are transmitted to the latter and may be recorded. This technique, termed ballistocardiography, has been applied to the study of cardiac abnormalities and to the estimation of cardiac output.

We shall terminate this series of examples with an illustration from the area of respiration. Figure 6 shows one form of closed-circuit respiratory calorimeter. This instrument is used to measure the oxygen consumption of a subject, which it does directly by indicating the decrease in volume of a mixture of oxygen and air as the subject rebreathes into the system. The carbon dioxide is absorbed as the gases pass over soda lime. From the oxygen consumption, a very good estimate of total heat production may be derived by simple arithmetic. This is the machine used to measure the basal metabolism of man or animals.

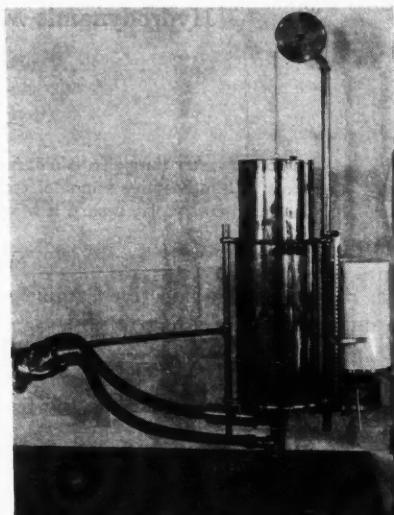


FIG. 6. Benedict-Roth respiration calorimeter. The subject rebreathes a mixture of air and oxygen, the carbon dioxide being absorbed by soda lime. Oscillations of the bell are recorded on the drum to the right, which also records the shrinkage in gas volume (oxygen consumption of the subject) during the test.

I have tried to indicate some of the simpler and more obvious applications of physics to medical studies in the hope that, as teachers of physics, you may find such information helpful. For the average physician, the physics requirements are at an elementary level, but they are definitely valid. We should like medical students to handle their physical problems and physical instruments intelligently. For this objective, they require a good grounding in classic physical principles, and such preparation is afforded by the usual first course. However, some barriers to learning must be removed. Perhaps some device as yet untried may convince students that physics is not solely for the edification of engineers and other physicists. In any event, little will be accomplished to correct the deficiencies which are now prevalent without the help of the teachers of physics themselves.

Central Pennsylvania Section

The Central Pennsylvania Section of the American Association of Physics Teachers will meet April 9-10, 1954, at *Juniata College*, Huntingdon, Pennsylvania. PAUL R. YODER, *Juniata College*, is chairman of the local committee on arrangements for the meeting.

Hydrodynamic Models of Radioactive Decay

HAROLD P. KNAUSS

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(Received August 7, 1953)

The flow of water through a capillary from a vessel with vertical walls is shown to simulate the decay of a radioactive sample if the velocity of efflux is low. The rate of flow, when both inertia and viscosity play a part, is worked out under simplifying assumptions.

THE decay of a radioactive sample of N atoms occurs at a rate given by

$$-dN/dt = \lambda N, \quad (1)$$

apart from statistical fluctuations, where the decay constant λ is the probability that any individual atom will decay within one second. This equation and its integral

$$N = N_0 e^{-\lambda t} \quad (2)$$

adequately describe radioactive decay, but their full significance is not immediately evident to those who lack experience with phenomena following this behavior. To help students visualize the implications of these equations, Bohn and Nadig¹ designed a hydrodynamic model, a vessel shaped so that the flow of water through a nozzle is proportional to the depth of water remaining. Here Torricelli's law applies,

$$v = (2gy)^{\frac{1}{2}}, \quad (3)$$

in which v represents the velocity of flow from the nozzle, g is the acceleration due to gravity, and y is the depth of water. The horizontal section of their vessel was made inversely proportional to the square root of the height above the bottom of the vessel. The observed behavior was in good agreement with Eqs. (1) and (2) with either mass or depth as the variable over a working range of more than four half-life periods. Depending on the nozzle dimensions, various half-life values of the order of a minute were obtainable.

Their models were designed primarily for demonstration but could be used also for student measurement with regard to decay constant, average life, secular and transient equilibrium, etc. The models are always ready for use, and

¹ J. L. Bohn and F. H. Nadig, Am. Phys. Teacher 6, 320-3 (1938).

they naturally are free from the risks associated with handling of actual radioactive materials.

A model based on capillary flow rather than Torricelli flow was demonstrated by the writer² at the Middletown meeting. It has the advantage that a vessel of uniform cross section is used instead of one constructed specially. According to Poiseuille's law, capillary flow is directly proportional to pressure difference, and therefore to depth. The law may be given the form

$$\pi R^2 v = (\pi R^4 \rho g y) / 8kL, \quad (4)$$

where R is the radius of the capillary, L is its length, k is the viscosity and ρ the density of the liquid, v is the average velocity in the capillary, and y is the head.

The model consisted of a 1000-ml cylindrical graduate with a siphon of glass capillary tubing bent into U shape. The full graduate was balanced on a platform balance to assist accurate timing by noting the instant when balance was restored after removing selected weights from the other pan. A color change was used to emphasize that a transformation was being illustrated, by tinting the water in the cylinder with an indicator that turned colorless in the presence of a few drops of acid placed in the glass collecting vessel.

Previous study of this model revealed that the behavior shown by a semilog plot of mass remaining against time was characterized by a line curving gently downward instead of the desired straight line. It was noticed that the water issued from the capillary with appreciable kinetic energy, indicating that inertia played a part along with viscosity in limiting the flow. By slipping a short length of rubber tubing on the end of the capillary, the area of cross section of the exit was increased and the velocity of

² H. P. Knauss, Am. J. Phys. 18, 521A (1950).

efflux was reduced enough so that a good straight-line graph was obtained over a range of about 7 half-life periods. The half-life was decreased by this modification from 4.2 to 3.5 minutes. A design incorporating a large orifice is illustrated in Fig. 1.

Since a low exit velocity is essential for correct behavior of the capillary model, the half-lives readily attainable are greater than those of the nozzle model, and the action presumably is less dramatic. One of the experimental difficulties is determination of the effective mass of the "sample" which may be either more or less than the mass of water actually present in the vessel initially, because the orifice may be somewhat below or above the bottom, especially when the effects of surface tension are taken into account. One method of establishing the effective mass is to measure the rate of efflux using fairly short time intervals, plotting the rates against the mass collected, and extrapolating to find the total mass corresponding to zero rate of flow.

The conditions determining the rate of flow in the intermediate condition when both inertia and viscosity affect the rate of flow, while not providing a good model of Eqs. (1) and (2), are of interest in their own right. If it is assumed that the liquid issues from the capillary into the open without change in average velocity or cross section, the governing equation may be derived as follows. Assume that the head y in the vessel is made up of two parts, y_1 from Eq. (3) and y_2 from Eq. (4). Physically, this is to say that part of the head is responsible for the kinetic energy of efflux, and the remainder for maintaining viscous flow. By adding y_1 and y_2 and solving for v , one obtains

$$v = \left(2gy + \frac{64k^2L^2}{R^2\rho^2} \right)^{\frac{1}{4}} - \frac{8kL}{R^2\rho}, \quad (5)$$

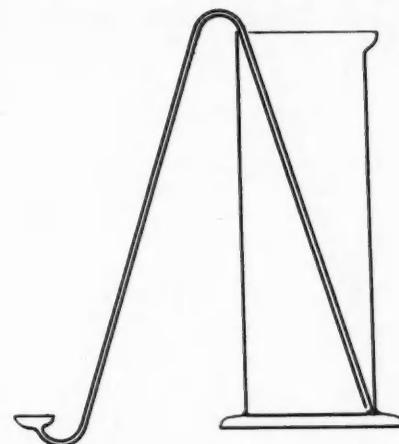


FIG. 1. Cylindrical graduate and capillary siphon with enlarged opening blown on delivery end to reduce velocity of efflux.

and the rate at which mass leaves the vessel is

$$-\frac{dm}{dt} = \pi \rho R^2 v = 8\pi k L \left[\left(1 + \frac{R^4 \rho^2 g y}{32 k^2 L^2} \right)^{\frac{1}{4}} - 1 \right]. \quad (6)$$

To show that this reduces to Poiseuille's law when viscosity predominates, note that if R is small in comparison with L , the second term in parenthesis is quite small so that the binomial theorem is applicable, to give, by neglect of terms beyond the second,

$$-(dm/dt)_{R/L \rightarrow 0} = \pi R^4 \rho^2 g y / 8kL, \quad (7)$$

which differs from Eq. (4) only because it refers to mass rate instead of volume rate of flow.

The relationship given in Eq. (6) must be interpreted as qualitative rather than exact, since the *vena contracta* effect is ignored in the derivation.

Southern California Section

The Southern California Regional Section of the American Association of Physics Teachers announces its **Tenth Annual High School Physics Test** to be given Saturday, May 1, 1954, at Compton College, Santa Barbara, University of Redlands, and elsewhere. Details may be secured from PROFESSOR COLLIS M. BARDIN, Chairman, Test Committee, *Compton College, Compton, California*.

Physics of the Glassy State. II. The Transformation Range*

E. U. CONDON

Corning Glass Works, Corning, New York

(Received July 14, 1953)

The concept of fictive temperature as a means of describing nonequilibrium states of glass is described. Activation energies and the modern views of chemical rate processes are discussed in relation to the kinetics of the approach of these nonequilibrium states to equilibrium.

1. FICTIVE TEMPERATURE

THE various physical properties of most crystalline solids change with time at a sufficiently rapid rate that they keep up with the rate of change of temperature occurring in usual situations. Thus if we heat up some zinc in a crucible and use a thermocouple to measure the temperature, as the temperature passes through the melting point in either direction, the metal will change quite promptly from the crystalline to the liquid condition. This behavior is the basis of a familiar technique in metallurgical studies, in which transformation points are located by irregularities on heating and cooling curves occasioned by abrupt absorption or evolution of latent heats of transformation occurring at these transformations.

However, the situation is quite different in silicate systems (crystalline as well as vitreous) where the time rates of transformation may be so sluggish as to require very long periods of time to bring about thermodynamic equilibrium.¹ In consequence, it frequently happens, in the laboratory and in industrial processes, that one is dealing with material that is not in internal structural equilibrium appropriate to its temperature. A proper approach to the description of the phenomena involved requires careful consideration of the kinetics of these slow approaches to equilibrium; that is, to the time and temperature relations.

Most glasses show interesting behavior of this sort in the general neighborhood of 400° to 600°C, which has come to be known as the *transformation range*.²

* The second of a series of lectures delivered at the Fifteenth Annual Colloquium of College Physicists, State University of Iowa, Iowa City, June 19, 1953.

¹ F. C. Kracek, *Phase Transformations in Solids* (John Wiley and Sons, Inc., New York, 1951), Chap. 9. See also M. J. Buerger's Chap. 6, of the same book.

Above the transformation range the rates of internal structural change are so great that, for the rates of change of temperature ordinarily encountered, internal changes are always essentially in equilibrium with the instantaneous temperature. Below the transformation range, these rates are so slow that they are negligible in periods of time like weeks, months, or even years. Therefore, at temperatures below the transformation range, the internal structure remains sensibly unaltered with time, even when the material is warmed or cooled or put under mechanical or electrical stress.

The particular internal structure thus frozen into the material will depend on its thermal history; that is, on the time-temperature cycle involved in bringing it down from high temperatures through the transformation range. Strictly speaking, the resulting structure probably depends on every detail of the time-temperature cycle, but it has proved useful to introduce a simplifying hypothesis, which is probably not strictly true, but which gives a good first approximation. The hypothesis is that the internal structure below the transformation range may be characterized by a single parameter τ , introduced by Tool and called by him the *fictive temperature*, (fictive for short) independently of

² Phenomena associated with the transformation range in glass already have a large literature. See W. A. Weyl, Chap. 11 of the book referred to in footnote 1, also J. E. Stanworth, *Physical Properties of Glass* (Clarendon Press, Oxford, 1950), Chap. 8, for general reviews and specific journal references and G. O. Jones, *J. Soc. Glass Technol.* **32**, 382 (1948). The basis for the modern approach was laid by A. Q. Tool of the National Bureau of Standards whose many papers should be consulted. A few of the most important of these are: (a) A. Q. Tool and C. G. Eichlin, *J. Opt. Soc. Am.* **4**, 340 (1920); A. Q. Tool and C. G. Eichlin, *J. Research NBS* **6**, 523 (1931). (b) A. Q. Tool and E. E. Hill, *J. Soc. Glass Technol.* **9**, 185 (1925). (c) A. Q. Tool and J. Valasek, *Sci. Papers Bur. Standards*, No. 358. (d) Tool, Lloyd, and Merritt, *J. Am. Ceram. Soc.* **13**, 632 (1930). (e) A. Q. Tool, *J. Am. Ceram. Soc.* **29**, 240 (1946).

the details of the time-temperature cycle by which the final state was produced.

On this view the condition of a glass at temperatures below about 600°C is characterized by two temperatures, the usual T , which is what a thermometer in contact with the material reads, and τ , the fictive which characterizes the frozen-in internal structure as determined by the previous thermal cycle of the material.

Operationally the fictive τ is defined as follows: for temperatures T , below the transformation range, the material is said to have a fictive τ if it is in the *state* produced by rapid quenching from complete equilibrium at temperature $T = \tau$. The statement should be made more precise by saying how the *state* is to be characterized; that is, what measurable physical property is to be used to determine the fictive temperature scale. Density or linear expansion of the glass are commonly used as will appear in detail later.

Since rapid quenching will necessarily occur at a finite rate, the resulting structure will always correspond to a somewhat lower fictive temperature than that from which the quenching took place. This effect becomes more pronounced in quenching from higher temperatures where transformation rates are highest. Because quenching necessarily occurs at a finite rate, there will be a practical *upper* limit to the values of the fictive τ , which can be attained. Also because complete equilibrium at a temperature T requires longer and longer times for its establishment as T becomes lower, there will be a practical *lower* limit to the attainable values of the fictive τ . The transformation range is the more or less inexactly defined range between these practical upper and lower limits.

These ideas may be made more clear by reference to Fig. 1₂, an idealized plot of the total fractional expansion in length (thousandths or mils) from 0°C to the temperature shown as abscissas. The steeply sloping line (a) in the range $T > 500^{\circ}\text{C}$ is the expansion of the glass when held long enough at each temperature to be in complete equilibrium. Above 600°C the rates of internal structural change are so fast that we would have to quench a small specimen from somewhat more than 600°C to put it in a condition such that for $T < 400^{\circ}\text{C}$, its expansion varies along the line $\tau = 600^{\circ}\text{C}$, which corre-

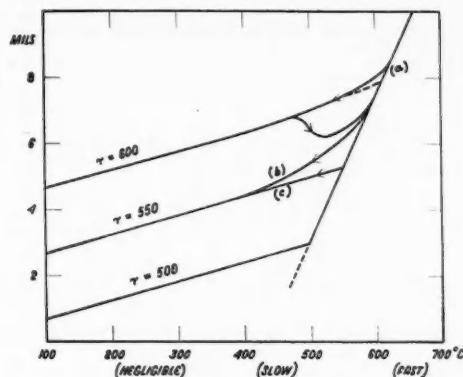


FIG. 1₂. Idealized linear expansion plot in the transformation range.

sponds in slope to the low temperature coefficient of expansion and in location is the line which intersects (a) at $T = 600^{\circ}\text{C}$.

Suppose the cold glass has a fictive of 600°C and that it is warmed up *very slowly*. It expands along the $\tau = 600$ line until, somewhere near 500°C , the rates of internal transformation become appreciable, permitting the fictive τ to decrease. This may produce a greater contraction than the expansion due to increasing T , with the result that the linear dimensions go through a maximum followed by a minimum, as indicated.

For an appropriate rate of cooling from equilibrium at $T = 575^{\circ}$ the curve might be like the one marked (b) joining asymptotically with the line $\tau = 550$, which corresponds to a rapid quench from equilibrium at $T = 550^{\circ}\text{C}$, indicated in curve (c). The basic hypothesis is that for $T < 400^{\circ}\text{C}$ the properties of the glass will be the same, whether the actual thermal history was along (b) or along (c). Thus different thermal histories may lead to the same τ and, if they do, in this approximation the properties will be the same.

The working value of the hypothesis, that the thermal history is adequately described by the single parameter τ will depend on the extent to which values of τ are the same when different measurable properties are used to establish the fictive temperature scale. This question has not been studied as critically up to now as one would like, so we shall proceed to make the under-

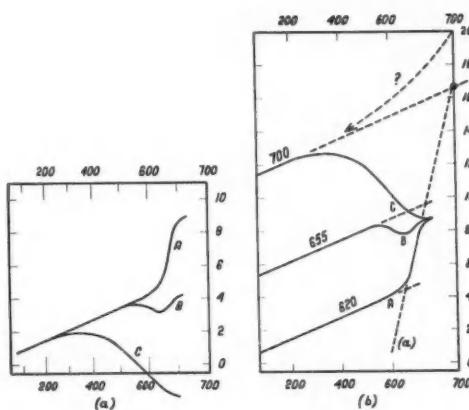


FIG. 2. Expansion in borosilicate glass (after Peychès).

lying hypothesis and go on, bearing in mind that it may need elaboration when more careful studies are made.

An excellent example of the qualitative behavior indicated in Fig. 1₂ is shown in Fig. 2₂, part (a) of which is taken from Peychès.³ Ordinates are the total fractional increase in length in thousandths of three pieces of the same glass, a borosilicate, on being warmed up from room temperature. *A* refers to a carefully annealed sample, *B* refers to a rapidly chilled massive sample, and *C* refers to a much more rapidly chilled fiber, 6μ in diameter.

According to the point of view presented in the foregoing discussion, it is more natural to regard the glasses as being in the same structural condition when *above* the transformation range, and so to displace the three curves vertically with respect to each other until they coincide at, say, 650°C, producing the result shown in part (b) of Fig. 2₂. The dotted lines have been sketched in to correspond with Fig. 1₂. The steep dotted line (a) corresponds to (a) in Fig. 1₂ and represents the expansion when complete equilibrium is established. As *A* was carefully annealed the glass presumably followed along (a) when it was prepared, to $T=620^\circ$, and then broke away to contract on the line of gentler slope, the same as the line on which expansion was observed. Had the annealing been still more "careful," that is, slower in the range 600–650°C, it would have

been possible to follow (a) farther down, producing a still lower fictive in the glass. But the transformation rates become so slow at this end of the range that the cooling rate has to be reduced enormously to produce a slight lowering of fictive. Thus 620°C is essentially the practical lower limit of the transformation range for this glass. For *B* the fictive is 655°. This glass on warming up in the expansion apparatus expanded because of increasing temperature, went through a maximum at 580°C and then began to contract because the fictive is cooling at a rate which more than counteracts the expansion due to increasing temperature. Curve *C* shows the same behavior in an even more extreme form. Presumably when it was originally formed it traversed some such curve as the sketched one marked "?" and ended with a fictive of approximately 700°C which is so high that it started to contract, on being warmed up, at a temperature somewhat below 400°C.

Evidently one of the most important questions we can ask is: what are the laws governing the temperature dependence of the rates of these structural changes in the transformation range?

2. ACTIVATION ENERGIES AND TIME RATES

Throughout the whole of the modern theory of physico-chemical rate processes, no matter how or by whom the details are formulated,⁴ the dominant factor in the temperature dependence

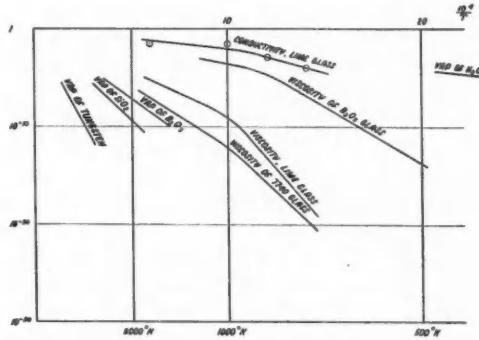


FIG. 3. Arrhenius function.

⁴ Glasstone, Laidler, and Eyring, *Theory of Rate Processes* (McGraw-Hill Book Company, Inc., New York, 1941), is a good general presentation of modern application of statistical mechanics to rate processes.

³ I. Peychès, J. Soc. Glass Technol. 36, 164 (1952).

is the Arrhenius function

$$e^{-E/RT} = 10^{-A/T},$$

where T is the absolute temperature, R the gas constant in cal/mole degree, and E the activation energy in cal/mole. The constant A is in degrees and what is derived directly from the slope of a plot of $\log_{10}(\text{rate})$ against $1/T$. It is related to E by

$$E = 4.57A \text{ cal/mole.}$$

The Arrhenius function varies over enormous ranges in various physico-chemical phenomena such as the vaporization of helium and the vaporization of tungsten (vaporization can be considered as a rate process or as an equilibrium—in fact, vapor “pressure” is often measured by the rate of loss of weight of an evaporating solid). It is therefore important to become familiar with the values of E for various processes and the associated behavior of the Arrhenius function. Figure 3₂ shows the log of the function plotted against the reciprocal of the absolute temperature. On it are plotted the Arrhenius functions governing the vaporization of several substances, in the range of temperature in which they are usually considered, also the curves for the function governing the viscosity of a lime glass and of B_2O_3 , and also a curve for the temperature dependence of the electrical conductivity of the same lime glass whose viscosity variation is plotted.

These curves have been displaced vertically parallel to themselves in such a way that their high-temperature portions extended would pass through the origin. The interesting special feature about the curves for glass is that they are not straight, bending in such a way as to have considerably greater slope in their low-temperature portions, that is, they show higher activation energies in the low-temperature range.

Every student of physico-chemical rate processes will find it highly instructive to collect his own private file of experimental values of activation energy. A start toward this is afforded by Fig. 4₂, in which the values⁵ of E from 1 to 200

⁵ The diffusion values are from F. Seitz, Chap. 4 of the book cited in footnote 1, as are the viscosity and electrical conductivity data used here and in Fig. 3. The activation energy for 7740 glass is based on viscosity data from the Corning Glass Works bulletin, “Properties of Selected

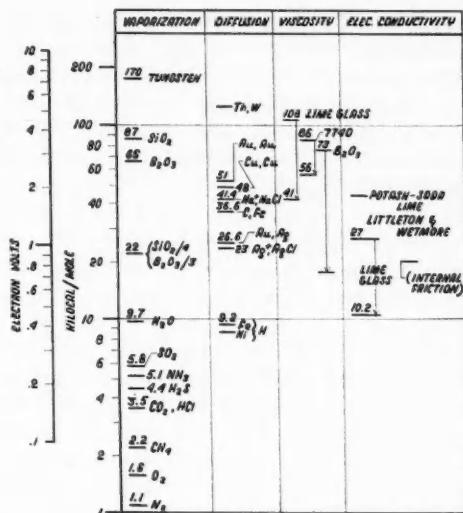


FIG. 4₂. Some activation energies.

kcal/mole are laid out vertically on a logarithmic scale for a variety of processes. The range of values for the glasses is indicated, high values referring to the low-temperature regime, and low values to high temperatures.

Let us now have a very general look at physico-chemical rate process theory. The observed macroscopic rate, whether it be of diffusion, of shear flow, of transport of electricity, of evaporation, of gas phase chemical reaction, of oxidation of metallic surface, is the net statistical effect of many elementary atomic or molecular processes. A given process will be called a *mechanism* and will be called *forward* if its direction contributes to the observed macroscopic rate, and *backward* if it works against this rate. Both forward and backward processes are going on all the time. In physico-chemical equilibrium they go on at equal rates in both directions (principle of detailed balancing). Macroscopically observed rate phenomena are the resultant of usually an extremely slight unbalance of the two rates, ex-

Commercial Glasses,” Corning, 1949. The viscosity and heat of vaporization data for B_2O_3 are from M. P. Volarovitch and D. M. Tolstoi, J. Soc. Glass Technol. 18, 209 (1934) and G. S. Parks and M. E. Spaght, Physics 6, 69 (1935). The value $E=20$ labeled “internal friction” is for the temperature dependence of internal friction of a commercial plate glass measured by J. V. Fitzgerald, J. Am. Ceram. Soc. 34, 388 (1951).

pressed as a fraction of either of the two nearly equal rates.

The first step in building a rate theory consists in recognizing the possible mechanisms which might be contributing. The next step involves application of statistical mechanics to the forward and backward rates. This involves calculation of the relation between the macroscopic rate parameter and the net frequency of occurrence of forward processes. It involves calculation of the way in which an external influence like applied stress or applied electric field affects the unbalance of these rates. It involves a calculation of the rates of forward and backward processes by application of statistical mechanics, making use of the fact that the unbalance is small, so the departure from thermodynamic equilibrium will be small enough to validate equilibrium distributions of energy among atomic coordinates.

In processes governed by an activation energy factor the common element is that some atomic or molecular configuration must pass from a state A to a state B in the forward process, and from B to A in the backward process, and that transition between these states is only possible when the energy in the coordinate which describes motion from A to B exceeds some minimum activation energy E . Thus the factor $e^{-E/RT}$ enters as the fraction of all possible motions in this coordinate which meet this condition.

The unstable configuration between A and B through which the system must pass is called the *activated complex*. Only a certain fraction of the events, even if the energy is sufficient, may be successful in bringing about a passage, $A \rightarrow B$. For example, suppose we are considering the kinetics of the reaction



First we may adopt effective sizes for the molecules and calculate the total number of collisions in unit time in unit volume. Then the effective number leading to forward reaction will be reduced, not only by the Arrhenius factor, but also perhaps in that only collisions which meet some other conditions of proper mutual orientation at the instant of collision or the like, can lead to reaction.

It has recently become the practice to write this additional factor (usually unknown in any detail) in the form $e^{S/R}$ and speak of S as the entropy (per mole) of the activated complexes, relative to the initial reactants in the free state. Thus if N is the number of collisions of all sorts in unit time per mole of reactants, the forward rate in terms of number of forward processes in unit time per mole is

$$f = Ne^{S/R} e^{-E/RT}$$

or

$$f = Ne^{-F/RT},$$

where

$$F = E - TS$$

and is called the *free energy of the activated complex*, relative to the initial reactants.

In the foregoing highly simplified summary it was implied that only a certain *fraction* of the collisions having requisite energy might be successful implying $e^{S/R} < 1$, or a negative entropy of activation. In cases in which the reactants are large molecules with many internal degrees of freedom, it may happen that the activated complex is less restricted by quantization conditions than the initial reactants which means that S is positive and $e^{S/R} \gg 1$. This is especially true in the kinetics of important biochemical reactions.

This summary is a totally inadequate account of the subject, but it is hoped that it can serve to make a study of the literature easier by highlighting the structure of the argument that is common to all theories of this kind.

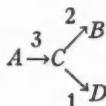
Suppose now that there are several distinct alternative mechanisms $A_1 \rightarrow B_1$, $A_2 \rightarrow B_2$, etc., which can produce the observed macroscopic result. If these are on an *either-or* basis then the total rate will be the sum of the rates by each of the mechanisms. Because these rates vary so enormously with changes in the free energy of activation F , in the factor $e^{-F/RT}$, it will usually happen that the mechanism of the fastest rate will be overwhelmingly fast compared to the others so that the whole rate is essentially that of the fastest mechanism.

If the free energy of activation of two mechanisms was equal for a particular temperature, (as indicated in Fig. 5, where the two straight lines represent $\log(e^{-F/RT})$ for the mechanisms 1

and 2, respectively, shown intersecting at 1200°K). At higher temperatures 1 is the faster mechanism and will dominate the rate, while at lower temperatures 2 is the faster and dominates. The two mechanisms may be said to be "in equilibrium" or contribute equally to the rate when each has the same free energy of activation for its activated complex.

Such a result gives an understanding of a situation in which the $\log(\text{Rate})$ against $1/T$ curves have lower activation energy at the low-temperature end. But we have seen in Fig. 3₂ that the bend in such curves for glass viscosity is in the opposite sense!

If the bend in the curves for glasses is to be interpreted as a change-over between different mechanisms one has to devise a way of coupling the two in series, so that the over-all rate is dominated by the slower of the two whose joint operation is required for the total effect. The logical "and" must connect them instead of the "or," as indicated on Fig. 5. Various possibilities are open. Thus suppose transitions go by way of a stable intermediate C



and each process has its own free energy of activation as indicated by the subscripts. Here it

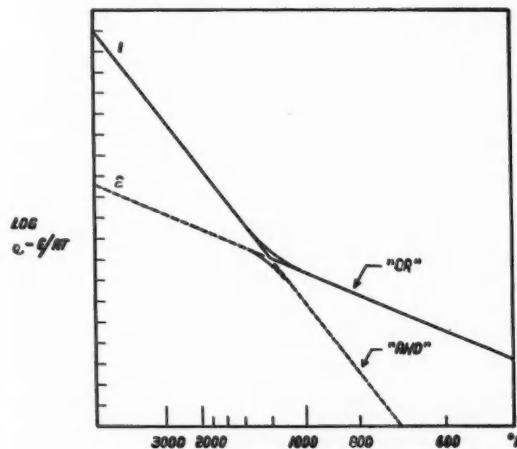


FIG. 5₂. Change-over between mechanisms.

is supposed that $C \rightarrow D$ represents a loss or "quenching" of C's so they can no longer contribute to the process which is only aided by processes ending at B. Write f for F/RT . The net rate for $A \rightarrow C$ is $N e^{-f_3}$ and for $C \rightarrow B$ and $C \rightarrow D$ it is $N' e^{-f_2}$ and $N' e^{-f_1}$ with N' determined by $N e^{-f_3} = N' [e^{-f_1} + e^{-f_2}]$ giving as the over-all rate of arrival to B

$$r = N \frac{e^{-(f_2+f_3)}}{e^{-f_1} + e^{-f_2}}.$$

At high temperatures $e^{-f_1} \gg e^{-f_2}$ as in Fig. 5₂, so the rate has an apparent activation energy $(E_2 + E_3 - E_1)$, while at low temperatures the inequality is in the opposite sense so the rate has an apparent activation energy $(E_2 + E_3 - E_2)$. Since by hypothesis (from the way the curves are drawn in Fig. 5₂) $E_1 > E_2$, it follows that on this model there is a lower effective activation energy at high temperatures than at low, the behavior shown by glass.

What is lacking is a more specific picture, in terms of the random network model, of exactly how such a two-stage mechanism operates.

3. DEPENDENCE OF OTHER PROPERTIES ON THE FICTIVE

Below the transformation range the glass remains with a frozen-in structure characterized by its fictive τ . On being cooled down rapidly it contracts but not nearly as much as if cooled down slowly, as Fig. 2₂ clearly illustrates. With further drop in temperature it does contract somewhat, this being the expansion ordinarily measured as the usual reversible thermal expansion at temperatures below 400°C.

Adopting linear approximations, the glass has an equation of state in which its specific volume is a function both of its temperature and its fictive

$$V(\tau, T) = V_0 [1 + 3\alpha'(\tau - \tau_0) + 3\alpha T].$$

Here α is the usual coefficient of linear expansion for the glass, V_0 is the volume at zero degrees temperature and at least practicable fictive τ_0 , while $V(\tau, T)$ is the specific volume at temperature T and fictive τ . Evidently α' is the analog of α , the fractional change in length per unit change in fictive. Generally α' is about three

times α , but the total amount of data capable of being treated in this way is not extensive. The density is given by

$$\rho(\tau, T) = \rho_0 [1 - 3\alpha'(\tau - \tau_0) - 3\alpha T].$$

If the constants are known, a measurement of density serves as a fictive thermometer.

In the foregoing it is tacitly supposed that the specimens are small enough, relative to the rate of temperature cycling to which they are exposed, that all parts of the specimen have the same thermal history. Otherwise the material will not be in the same condition throughout. It will have fictive gradients in it. This situation is an important source of frozen-in stress distributions occurring in glass articles which have not been fully annealed.

Refractive index is extremely closely related to density and so may be calibrated instead of density as a fictive thermometer. Some recent unpublished measurements of H. N. Ritland made in the Research Laboratory of Corning Glass Works on the relation of density and refractive index for Glass 8370, an optical glass, showed the relation to be linear to high accuracy with $dn/d\rho = 0.167 \text{ cm}^3/\text{g}$, with the experimental uncertainty in each density value being about $\pm 10^{-6} \text{ g/cm}^3$ and each refractive index value being good to $\pm 20 \times 10^{-6}$. The observed $dn/d\rho$ is definitely less than that to be expected if the atomic refractivities remained constant, that is if ρ were proportional to $(n^2 - 1)/(n^2 + 2)$. The departure is in the sense of a slight drop in atomic refractivities with increasing density.

Tool^{2a} made qualitative observations by the method of differential thermal analysis of the heat effects associated with the transformation range. Apparently no accurate calorimetric work has been done. His method was to heat the glass in a furnace whose temperature was increased at a steady rate of 6 deg/min, having one junction of a thermocouple in the glass and the other in a neutral body. If there is no change with temperature of the heat capacity of either then the temperature difference will remain constant as the two bodies heat up together, although the actual temperature of one may lag behind the other by a constant amount. If now the glass gives off some heat in a certain range, its temperature will run ahead of that of the neutral body, while if it

absorbs heat it will run behind. Effects of only a few degrees are easily noted but not evaluated in terms of heat units.

His results are consistent with the general picture that the heat content depends on the fictive as well as the temperature,

$$H(\tau, T) = H_0 + c'(\tau - \tau_0) + cT,$$

in which c' is a fictive heat capacity and c the usual heat capacity. Below the transformation range τ is constant so the effective heat capacity is c , but above the range $\tau = T$ so the effective heat capacity is $(c' + c)$. At present it is not known whether such a linear expression for $H(\tau, T)$ is adequate. The results of Thomas and Parks on the heat capacity of boric oxide alluded to in Part I indicate that additional heat changes occur in the transformation range.

One would certainly expect various other properties to depend on τ as well as on T . The available information on elastic properties at high temperatures is rather meager and the literature is confused by the lack of definite specifications of heat treatments used. The most careful work is that of Stong⁶ who measured Young's modulus E on a soda-lime glass. He found $E = 7600 \text{ kg/mm}^2$ at 0° , which decreased to 7000 at 400°C and thereafter decreased even more at a rapidly increasing rate. He found that the curves were displaced more or less parallel to themselves by amounts up to 300 kg/mm^2 for different thermal histories, indicating a dependence on the fictive.

That the electrical resistance of a soda-potash-lead glass depends on the fictive as well as on the temperature is clearly shown in results of Littleton and Wetmore.⁷ The main results are shown in Fig. 6, adapted from their paper. Ordinates are log of conductivity and abscissas are $1/T$. The long straight line marked "equilibrium" is presumably what one would obtain if it were possible to anneal long enough to bring τ down to T , although their experimental points do not go below $T \sim 370^\circ$ along this curve. The broken curve marked "chilled" represents the course of the conductivity at lower temperatures for samples rapidly cooled, whose fictive there-

⁶ G. E. Stong, J. Am. Ceram. Soc. **20**, 16 (1937).

⁷ J. T. Littleton and W. L. Wetmore, J. Am. Ceram. Soc. **19**, 243 (1936).

fore remains high, perhaps $\tau \sim 430^\circ$, as T is dropped on down to 300° .

The frozen-in structure distends the network, making it easier for the ions, presumably mostly Na^+ , to move about, hence the effect of higher fictive is to increase the conductivity. The bend is in the opposite sense of those discussed in the preceding section for viscosity, at higher temperatures, and of a type compatible with *additive participation* of alternative mechanisms. The view then is this: in the open structure of high fictive some of the Na^+ ions can get through with an average activation energy of 27.4 kcal/mole. A good many more can get through a chain of higher hurdles requiring an average activation of 44.2 kcal/mole. At high temperatures this larger number dominates the conductivity, while at low temperatures, in equilibrium, the open structure closes up so the conductivity varies along the $E=44.2$ line. But in the chilled specimens the smaller number of ions which have a path of low hurdles open to them dominate, their fewer number being more than made up for by the extra ease with which they can move. This interpretation leads however to an extended and gradual transition between the intersecting straight lines, instead of to the rather sharp break reported by Littleton and Wetmore.

4. RATE OF CHANGE OF FICTIVE

At any temperature the vitreous equilibrium condition is that in which the fictive is equal to the temperature. Tool⁸ has set up a semiempirical law for the rate of change of the fictive with time and tested it against experimental data.

The first requirement is that $d\tau/dt$ be equal to a function of τ and T which vanishes for $\tau=T$ and which tends to diminish $|\tau-T|$. The simplest assumption is that $d\tau/dt$ is proportional to $(T-\tau)$, a kind of Newton's law of cooling for the fictive. However, the proportionality factor, whose reciprocal is a time constant measuring the time lag of τ behind T , is itself expected to be strongly temperature dependent.

One expects this to depend both on T and on τ and probably mainly through an Arrhenius

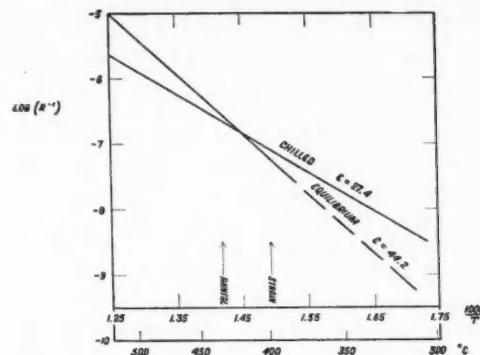


FIG. 6. Effect of fictive on electrical conductivity.

function $e^{-E/RT}$, but Tool uses a different form, one suggested by Twyman⁹ which is equivalent to it for the fairly small ranges of temperature involved. It consists of approximating to $Ae^{-E/RT}$ with the form $Ke^{T/k}$. Near to T_0 this is equivalent to an effective activation energy of $E=RT_0^2/k$. Tool also assumes a similar variation of the rate with τ , with a different effective activation energy. He thus postulates that

$$\frac{d\tau}{dt} = K \exp\left(\frac{T}{k} + \frac{\tau}{h}\right) (T - \tau),$$

which may be called Tool's law of fictive cooling. He shows that it gives a good account of his observations although some discrepancies are found.

Consider the change of τ with time when T is held constant. This would give the familiar exponential decay of $(\tau-T)$ if the coefficient of $(T-\tau)$ were constant. A few transformations reduce the Tool equation to standard form:

$$\theta = K \exp\left[T\left(\frac{1}{k} + \frac{1}{h}\right)\right] t \quad u = \frac{\tau-T}{h} \quad \frac{du}{d\theta} + e^u u = 0.$$

The solution involves different functional forms according as u approaches zero through positive values, or through negative values, because of the temperature dependence of the rate constant. In terms of the exponential integral functions,¹⁰

⁹ F. Twyman, J. Soc. Glass. Technol. 1, 61 (1917).

¹⁰ Jahnke-Emde, *Tables of Functions* (Dover Publications, New York, 1943), p. 6.

⁸ A. Q. Tool, J. Research Natl. Bur. Standards 37, 73 (1946).

the two forms are:

$$\begin{aligned} \text{for } u > 0 \quad \theta &= -Ei(-u) + Ei(-u_0); \\ \text{for } u < 0 \quad \theta &= -\bar{E}i(-u) + \bar{E}i(-u_0). \end{aligned}$$

It is natural to plot θ as abscissas and u as ordinates on a logarithmic scale because this gives a straight line plot in the case of exponential decay, appropriate to a constant rate. Tool's law also gives a straight line on this plot for $u \gg 1$, as then the variation of the rate is unimportant.

The theoretical curves are shown on Fig. 7₂. Curve (a) is for $u > 0$, for fictive cooling from above. Large values of u decay with great rapidity relative to the natural time unit of the problem. Curve (b) is for $u < 0$, fictive warming up from below. In the upper left are tabulated the abscissa values at which the curve crosses successive ordinates up to $u = 10$, indicating the extreme slowness with which the fictive warms up if its initial value is much below the temperature. The extreme rapidity for cooling and the extreme slowness for warming up is what defines practical limits to the transformation range, as remarked earlier.

A careful study of the Tool equation in relation to fictive change at constant temperature in 8370 glass was made by Ritland¹¹ in the Research Laboratory of Corning Glass Works during the past year. He finds his data definitely require a correction term, so he proposes a modified Tool equation

$$\frac{d\tau}{dt} = Ae^{T/k}e^{\tau/h}(T-\tau)[1+B(T-\tau)],$$

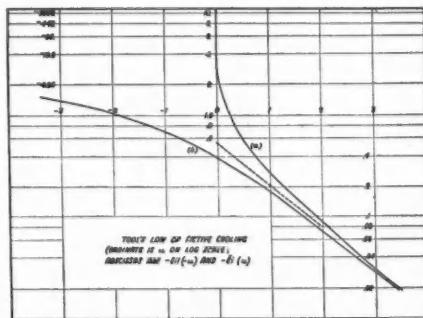


FIG. 7₂. Tool's law of fictive cooling (ordinate is u on log scale; abscissas are $-Ei(-u)$ and $-\bar{E}i(u)$).

¹¹ H. N. Ritland, J. Am. Ceram. Soc. (to be published).

with

$$\begin{aligned} A &= 2.12 \times 10^{-3} & B &= 0.244 \\ k &= 9.3^\circ\text{C} & h &= 36.4^\circ\text{C} \end{aligned}$$

This gives an excellent fit for the approach to equilibrium as observed both from above and from below three different constant soaking temperatures, $T = 520, 526$, and 533°C at which times of the order of 10 hours (at 520°C) down to 1 hr (at 533°C) were required to bring the glass essentially to complete equilibrium, that is, to $|\tau - T| < 0.5^\circ\text{C}$. The values of k and h expressed as activation energies in exponents E/RT and $E'/R\tau$ give rather low values, $E = 15.5$ and $E' = 3.5$ kcal/mole.

As already stated all of this work is on a semi-empirical basis. No attempt at a detailed kinetic rate process model for change of the fictive has been made.

5. COMPACTION OF GLASS BY HIGH PRESSURE

Some interesting new experimental results on a quasi-permanent compaction of glass under extremely high pressures have just been reported by Bridgman and Simon.¹² When fused silica and vitreous B_2O_3 are put under pressures of the order of 10^5 to $2 \cdot 10^5$ atmospheres, the material is compacted so that it shows a density increase of some 7 or 8 percent even after the pressure is released.

In order to achieve such great stresses the specimens were squeezed between two Carbonyl dies so the pressures were not exactly hydrostatic and were of somewhat uncertain amounts.

Their main results are shown in Fig. 8₂. The shapes of the curves are quite different which may well be correlated with the very great differences in the transformation temperatures, melting points, and so on of the two materials.

One might be inclined to think that the applied pressure simply acts to speed up the rate of transformation to the higher density form so that this proceeds even at room temperature to an extent that enables the effect to reach saturation in about a minute, the time the pressure was applied in this experiment. But the effect must be a more profound change than that. Extrapolation of known high- and low-temperature ex-

¹² P. W. Bridgman and I. Simon, J. Appl. Phys. 24, 405 (1953).

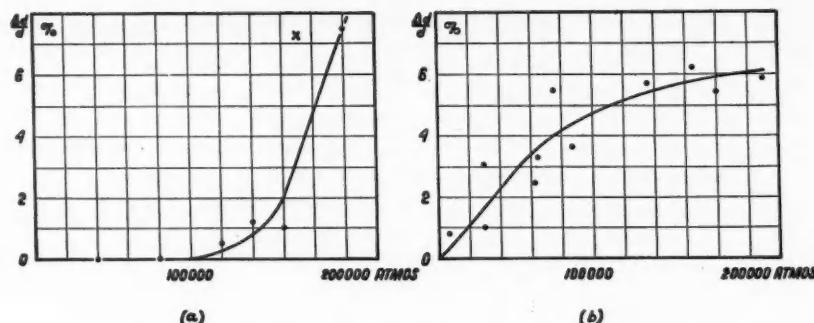


FIG. 82. Quasi-permanent density increase in (a) vitreous SiO_2 and (b) vitreous B_2O_3 as a function of applied pressure.

pansion coefficients for B_2O_3 indicate that there is only about 3- or 4-percent excess volume frozen-in in the glassy B_2O_3 structure at ordinary temperatures, whereas the compaction produced by 200 000 atmospheres is about twice this. There do not seem to be data available on extreme high-temperature expansion coefficients on fused silica itself which would permit estimation of the amount of the frozen-in excess volume in room temperature SiO_2 but it seems likely that this is not much more than about 4 percent either (using $\alpha = 5 \cdot 10^{-7}/^\circ\text{C}$, below the transformation temperature, assuming a fictive of 1500°C and that the expansion coefficient above the transformation range is about $15 \cdot 10^{-7}/^\circ\text{C}$). On the other hand the relative compaction involved in known crystalline forms of SiO_2 is adequate:

Form	cm^3/mole	Ratio
Vitreous	27.2	1.000
Tridymite	26.6	0.975
Cristobalite	25.9	0.953
Quartz	23.3	0.857

Thus spatial arrangements of the Si and O atoms whose volume is even smaller than the compaction due to $2 \cdot 10^5$ atmospheres are possible.

One may speculate that there is a molal volume V_p which is the equilibrium value for each pressure and that when the pressure p is applied and the volume is V the effect is to change the activation energy by an amount $-p(V - V_p)$ for forward compacting processes and by an amount $+p(V - V_p)$ for backward or expanding processes, therefore tentatively one

may expect an equation of the form

$$\frac{dV}{dt} = A e^{-E/RT} [e^{p(V-V_p)/RT} - e^{-p(V-V_p)/RT}]$$

as governing the rate of approach to equilibrium.

At room temperature $RT \sim 2.5 \times 10^4 \text{ cm}^3 \cdot \text{atmos}$ and the equilibrium volume change for $2 \cdot 10^5$ atmospheres is about $3 \text{ cm}^3/\text{mole}$ so initially $p(V - V_p)/RT$ is of the order 13 giving an enhancement of the rate by only e^{13} or 10^5 or 6 , which is not nearly enough to account for the effects observed relative to the rates of the ordinary transformation process. On the other hand an expression of the form of that suggested above can account for the enormous difference in rate observed on compacting and decompacting. Applying 40 000 atmospheres to vitreous B_2O_3 produced a density increase of 3.3 percent within about a minute. On release of the pressure the material starts to expand, that is, its density decreases but much more slowly as their Fig. 14 shows, the increase in density from its original value only having decreased to 2.2 percent after some 50 days, whereas the original compaction of 3.3 percent occurred in about one minute. The normal molar volume of B_2O_3 is $38 \text{ cm}^3/\text{mole}$, so the decrease produced in 1 minute by 40 000 atmospheres is about $1.2 \text{ cm}^3/\text{mole}$. It is not clear whether the value reached in one minute is the final equilibrium value, but it is probably within 10 percent of it, so the rate of compaction under high pressure is such that the change in the logarithm (to base 10) of $(V - V_p)$ is about 1 in 1 minute. With the pressure off (that is, reduced to 1 atmosphere) one sees roughly from

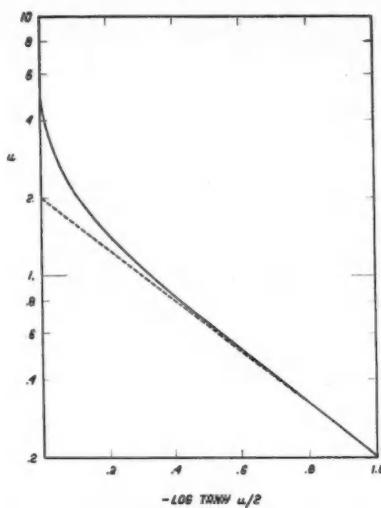


FIG. 92. Approach to equilibrium with generalized kinetic equation.

their Fig. 14 that a corresponding change requires of the order of 300 days or 4×10^5 minutes which is very crudely about the order of ratio of the rates to be expected under the two different pressures.

Much more experimental and theoretical study needs to be devoted to this interesting new phenomenon.

6. A GENERAL TRANSFORMATION RATE EQUATION

As already remarked, the rate factor for which Tool uses the form $e^{T/k}$ is more probably an approximation to $e^{-E/RT}$ valid only in a small range of temperature. But τ is itself proportional to the discrepancy between the actual volume and the equilibrium volume, and so probably belongs in the numerator of the exponential. The Bridgman-Simon experiments indicate that the rate of transformation, as well as the equilibrium condition, is pressure sensitive. All these things together suggest a general transformation rate equation.

The transformation may be characterized as going from configuration 1 to 2 and $E_1 S_1 V_1 \dots$ and $E_2 S_2 V_2 \dots$ are the molal thermodynamic quantities for these quasi-equilibrium configurations. To pass from 1 to 2 one must go through an activated configuration whose molal quantities will be written $E, S, V \dots$ without subscript.

The measure of the extent of the transformation is the change in volume ($V_1 - V_2$) related to x by

$$V = V_1 x + V_2 (1 - x),$$

and the transformation proceeds with change of x from 1 to 0.

The rates in each direction involve the Gibbs free energy of activation ($E - TS + pV$) relative to the initial and final states, hence the general kinetic equation will be assumed to be

$$\frac{dx}{dt} = -a \{ e^{-(G-G_1)x/RT} - e^{-(G-G_2(1-x))/RT} \}.$$

The rates balance, so equilibrium occurs, when $G_1 = G_2$, the familiar criterion. Writing

$$G_1, G_2 = \bar{G} \pm \frac{1}{2}g$$

the rate equation becomes

$$\frac{dx}{dt} = -2a \exp\left[-\left(\frac{G-\bar{G}}{RT}\right)\right] \sinh \frac{gx}{RT}.$$

At constant temperature and pressure this becomes

$$du/dt + A \sinh u = 0$$

with $u = gx/RT$ and $A = (ga/RT) \exp[-(G-\bar{G})/RT]$. This integrates to

$$At + \log \frac{\tanh(u/2)}{\tanh(u_0/2)} = 0.$$

The observed change in volume is related to u by the equation

$$v = V + (RTv/g)u.$$

This is qualitatively at least of the form observed for the dependence, starting off at a rapid logarithmic rate of transformation, and slowing down to the usual constant logarithmic decrement as equilibrium is approached, as shown in Fig. 92. Abscissas are values of $\log \tanh u/2$ on a linear scale, and ordinates are values of u on a logarithmic scale. For $u < 1$ the logarithmic decrement of u is essentially constant. The quantities $E_1 E_2, V_1 V_2, S_1 S_2, \dots$, depend on the variables p and T , which affects the kinetics of the process as well as the equilibrium.

At present the available data are insufficient for an adequate test of this general transformation rate equation.

Critical Reynolds Number and Flow Permeability

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(Received August 7, 1953)

Liquids of three different viscosities have been used in determining the flow permeability of fourteen media composed of spherical glass beads. Darcy's law has been found to hold for a Reynolds number up to 75. Three methods are given for determining the average bead diameters.

INTRODUCTION

THE flow permeability of a porous medium is a measure of the ease with which a fluid will flow through the medium. The fluid may be either a gas or a liquid.

Flow permeability is an important parameter in understanding many everyday phenomena, for example, the water seepage in earth-fill dams and irrigation ditches. Another example is the permeability of petroleum-bearing sands.

In determining the flow permeability of a porous medium in the laboratory it is essential that the flow velocity be kept below a certain upper limit; otherwise the flow breaks down into another state where (1) the velocity of efflux is no longer proportional to the pressure difference, (2) the values of velocity and pressure at a given point are not constant in time, as in the case of laminar flow, but oscillate about mean values, and (3) there is a continual intermixing of fluid elements.¹ This state of flow is called turbulence.

The initiation of turbulence is determined by the value of a dimensionless quantity called Reynolds number. For fluids in porous media, its value is given by

$$R = \delta v_e \rho / \mu, \quad (1)$$

where δ is an "average grain diameter," v_e is the true velocity through the pores (interstitial velocity), ρ is the density, and μ is the viscosity of the fluid.

Many workers² have shown that this precaution must be observed in order to avoid

appreciable deviations from Darcy's³ law, which states that the rate of flow Q of a fluid through a porous medium (horizontal filter bed) is directly proportional to the horizontal area and to the difference Δh between the fluid heads at the inlet and outlet faces, and is inversely proportional to the vertical thickness L of the porous bed; expressed analytically,

$$Q = c A \Delta h / L, \quad (2)$$

where c is a constant characteristic of the porous medium and of the fluid. More generally⁴ the flow velocity is given by

$$v = (k / \mu) (dp/dx), \quad (3)$$

where k is the flow permeability of the medium and dp/dx is the pressure gradient associated with the direction of the velocity v .

The authors have determined the flow permeability after the method of Terzaghi^{2,4,5} of fourteen media composed of nearly spherical glass beads using fluids of three different viscosities, namely, water, 10-centistoke and 100-centistoke Dow Corning silicone fluids. As the viscosity μ appears in the denominator of the expression for the Reynolds number, increasing the viscosity of the fluid reduces the Reynolds number. The more common unit of viscosity, the poise, is related to the centistoke through the density of the fluid ρ in the following way:

$$1 \text{ centistoke} = (1/0.01\rho) \text{ poise}. \quad (4)$$

This paper describes (1) the determination of the critical Reynolds number for flow permeability in media composed of spherical particles, and

¹ G. Joos, *Theoretical Physics* (G. E. Stechert and Company, New York, 1934).

² M. Muskat, *The Flow of Homogeneous Fluids Through Porous Media* (J. W. Edwards, Inc., Ann Arbor, Michigan, 1946).

³ H. Darcy, *Les fontaines publiques de la ville de Dijon*, 1856.

⁴ D. P. Krymire, *Soil Mechanics* (McGraw-Hill Book Company, Inc., New York, 1941).

⁵ Hydrostatic Pressure Method, ASTM Part IIIA, p. 1107 (1946).

(2) a comparison of three independent methods for determining average grain diameters.

METHODOLOGY

1. Determination of Critical Reynolds Number

The experimental method used by the authors for determining flow permeabilities was that of Terzaghi, which is based upon measurements of the rate of fall of the level of a fluid (under gravity) flowing through a vertical tube of uniform diameter packed with the porous medium. The equation relating the change in head to the flow permeability and other pertinent factors is, for this arrangement,

$$k = -(\mu L / \rho g t) \ln h_t / h_0, \quad (5)$$

where k is the flow permeability, L is the length of sample of porous medium, g is the acceleration of gravity, h_0 is the initial head ($t=0$), and h_t is the head at time t . The dimensions of k will clearly be those of an area (cm^2). It is customary in making use of Eq. (5) to plot the ratio h_t/h_0 against t on semilog graph paper; if such a graph does not yield a straight line, then obviously Eq. (5) cannot be used. A linear relationship will verify that the flow is not disturbed by a change in the medium, such as packing.

For an infinitesimally thin layer of bed, the interstitial velocity v_e of the fluid can be expressed in terms of the apparent velocity v (rate of fall of head) by⁶

$$v_e = v / \epsilon, \quad (6)$$

where ϵ is the porosity in percent. If L is the thickness of the bed and L_e the actual length of the tortuous path of the fluid, then the interstitial velocity of the fluid will be⁷

$$v_e = (v L_e / \epsilon L). \quad (7)$$

This is the expression used for the interstitial velocity v_e in Eq. (1) for the determination of Reynolds number.

The value used for L_e/L was that obtained from Kwong,⁷ where he gives $L_e/L = (2.5)^{\frac{1}{3}}$, or 1.58.

⁶ A. J. E. J. Dupuit, *Etudes Theoretiques et Practiques sur le Mouvement des Eaux* (1863).

⁷ J. N. S. Kwong, *et al.*, Chem. Engr. Prog. 45, 508 (1949).

2. Determination of Average Grain Diameters

For those media consisting of glass spheres all very nearly of the same size, one method of determining the average diameters was by means of micrometer measurements. A second method used for determining the average diameters of the spheres in these media and in mixtures made up of several sizes of spheres was derived as follows. For identically uniform spheres a constant can be determined by the equation

$$c = \eta \delta^3, \quad (8)$$

where η is the number of spheres per unit volume and δ is the diameter of the spheres. As the glass beads used in this experiment varied somewhat in diameter, δ used in Eq. (8) was the average of a number of measured diameters. The number of beads per cubic centimeter in the mixture was then determined by measuring the volume of beads resulting from mixing known volumes of beads of different sizes. The (root-mean-cube) diameter of the beads in the mixture was then determined by the application of Eq. (8) where now η is the number of beads per cubic centimeter in the mixture.

Equation (8) can be written

$$c = \frac{6\eta}{\pi} \left(\frac{1}{6} \pi \delta^3 \right) = \frac{6}{\pi} (\eta V), \quad (9)$$

where V is the volume of one sphere, and ηV is the total volume of the spheres per unit volume.

The porosity of the medium ϵ is introduced by the relationship

$$\epsilon = 1 - \eta V = 1 - \pi c / 6. \quad (10)$$

In the case of a mixture

$$\epsilon_{\text{mix}} = 1 - \pi c_{\text{mix}} / 6; \quad (11)$$

hence, in general,

$$\delta = [6(1 - \epsilon) / \pi \eta]^{\frac{1}{3}}. \quad (12)$$

The theoretical treatment of Kwong *et al.*⁷ was used as the basis of a third method of computing sphere diameters. Muskat² relates the two systems of permeability units (cm/sec and cm^2) by the equation

$$\bar{k} = k g \rho / \mu, \quad (13)$$

where \bar{k} is the permeability in cm/sec , k is the permeability in cm^2 , g is the acceleration of

TABLE I. Experimental results.

Medium No.	* Microm- eter	δ (cm) by		v_i (cm/sec) for		R H ₂ O	$k \times 10^8$ (cm ²)	
		Eq. (12)	Eq. (18)	H ₂ O	10 cs. Si.	100 cs. Si.	H ₂ O	10 cs. Si.
1.	37.5	0.0156	0.0161	0.0158	0.128	...	0.235	21.6
2.	37.5	0.0296	0.0302	0.0301	0.420	...	1.45	70.5
3.	37.5	0.0470	0.0476	0.0494	0.880	...	4.77	146
4.	37.5	...	0.0543	0.0575	1.05	...	6.50	178
5.	37.5	0.0575	0.0582	0.0610	1.19	...	7.90	201
6.	37.5	...	0.0624	0.0642	1.44	...	10.30	245
7.	37.5	0.0760	0.0782	0.0800	2.43	0.250	21.70	405
8.	37.5	0.1165	0.1189	0.1222	3.86	0.382	52.40	647
9.	35.0	...	0.1262	0.1290	5.22	0.530	75.00	825
10.	34.0	...	0.1359	0.1350	6.40	0.640	99.20	990
11.	33.0	...	0.1640	0.1600	7.55	0.905	0.092	141.00
12.	39.0	0.308	0.3176	0.3380	11.72	3.16	425.00	10.00
13.	36.0	...	0.3680	0.3620	...	7.12	0.708	0.103
14.	41.0	0.610	0.6220	0.6540	...	15.90	2.08	26.2
							99.0	1.30
								30 000
								39 041

gravity, ρ is the fluid density, and μ is the fluid viscosity.

Kwong *et al.*⁷ give the following equation for permeability:

$$k = \rho g \epsilon^3 / 5\mu S^2, \quad (14)$$

where ϵ = porosity of medium in percent and S = surface area of particles per unit volume.

By combining Eqs. (13) and (14) there results

$$k = \epsilon^3 / 5S^2. \quad (15)$$

From the relationship

$$s = \pi \delta^2, \quad (16)$$

where s is the surface area and δ is the diameter of a particle, it follows that

$$S = \pi \eta \delta^2, \quad (17)$$

where, as before, η is the number of particles per unit volume; hence:

$$\delta = (S/\pi\eta)^{1/2}. \quad (18)$$

The porosity of each medium was ascertained by the application of the relationship

$$\epsilon = 100(1 - \beta_s/\beta_b), \quad (19)$$

where β_s and β_b are the volumes of the spheres and the bulk volumes, respectively.

DISCUSSION OF NUMERICAL ANALYSIS

In Table I are listed in order the porosity in percent, the average sphere diameters by three different methods, the interstitial velocities, the Reynolds numbers, and the permeabilities. The porosities were determined by the application of

Eq. (19). In making these determinations it is necessary that the diameter of the graduated cylinder used for measuring volumes of batches of beads be large in comparison to the bead diameter. All of these values are for random packed spheres, the type of packing used for the permeability determinations.

Average diameters as determined by the three different methods are given. The micrometer method was used for media composed of spheres of one size, while Eqs. (12) and (18) were used for spheres of one size and for mixtures of these spheres. It is to be noted that in every case the diameter as given by micrometer measurement is slightly less than the diameters as given by Eqs. (12) and (18). This is because of the tendency to measure the smaller diameter in the case of quasi-spheres. The excellent agreement of the three methods indicates the validity of the application of Eqs. (12) and (18).

The interstitial velocities, as obtained from Eq. (7), are given for each of the media. Values are listed for water ($\mu = 0.00874$ poise, $\rho = 0.987$ g/cc), 10-centistoke silicone fluid ($\mu = 0.094$ poise, $\rho = 0.940$ g/cc), and 100-centistoke silicone fluid ($\mu = 0.968$ poise, $\rho = 0.968$ g/cc). Because of the low rate of flow of the higher viscosity fluids, it was not feasible to use these for permeability determinations of the media composed of the smaller diameter spheres. Because of its low viscosity no attempt was made to use water in determining the permeability of media Nos. 13 and 14.

The Reynolds numbers and values of flow permeability given in Table I were calculated by

means of Eqs. (1) and (5), respectively. By making flow permeability determinations, using fluids of different viscosities, it is possible to establish an upper value of the Reynolds number for consistent values of the flow permeability.

Table I indicates that essentially equal values of permeability can be obtained for a rather large range of Reynolds numbers. In one case (Medium No. 9) there is excellent agreement for a Reynolds number of 75. In every case there were large discrepancies in the value of the flow permeability when the Reynolds number reached values of approximately 100.

CONCLUSION

The authors suggest that several valuable laboratory experiments might be devised from the procedures outlined here. For example, an experiment on determining the average diameter of the beads in a mixture of several sizes of beads should be interesting and be informative in making use of the permeability method of obtaining surface area. Also, fluids of different viscosities might be used for determining the upper value of Reynolds number for laminar flow (consistent values of permeability).

NOTES AND DISCUSSION

Capacitance and Voltage Relationships*

WALLACE A. HILTON
William Jewell College, Liberty, Missouri

IN the general physics course, $Q = CV$ is a familiar equation. Apparatus to demonstrate this relationship of capacitance and voltage is shown in Fig. 1. A small power supply provides a potential difference of about 400 volts across a parallel-plate capacitor.¹ Immediately after the power supply is disconnected, the plates are separated. As the capacitance decreases, the voltage across the plates increases. In this case the increase is usually up to as much as 1200 volts, and will vary depending upon the time constant of the circuit and elapsed time between the disconnecting of the power supply and the separation of the plates.

The moisture content of the air seems to have some effect upon the resistance of the dielectric. A sheet of exposed x-ray film has proved satisfactory for the dielectric and does not seem to be affected by high relative humidity as much as some others that were used.

All of the switching is controlled by a motor, which when operated continuously repeats the cycle each 12 seconds. Small pilot lamps are used to indicate (1) when the power supply is connected across the plates, (2) when the power supply is disconnected, and (3) when the plates begin to separate. The voltage across the plates is read on a Heath² vacuum-tube voltmeter with high voltage probe.

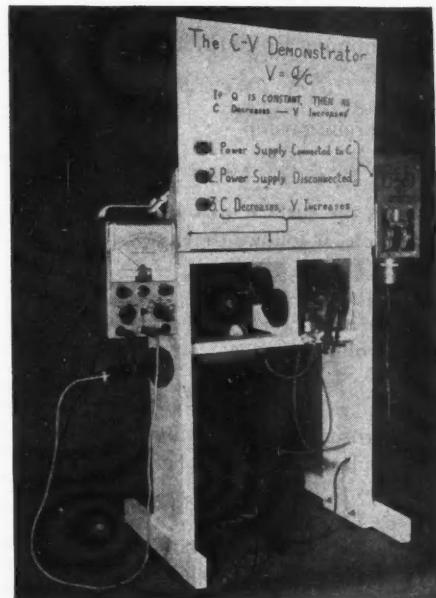


Fig. 1. The apparatus is standing on a table. The capacitor occupies a central position between the two supports. The upper plate of the capacitor is raised and lowered by a small wire attached over a bolt that protrudes from the rim of a rotating wheel. A schematic diagram is attached to the power supply which is located on the upper right side of the stand. The vacuum-tube voltmeter and the high-voltage probe are supported on the upper left side of the stand. The panel lights which indicate the various switching arrangements are shown in the upper center.

* A prize winner at the 15th Annual Colloquium of College Physicists, State University of Iowa, June 17-20, 1953.

¹ Central Scientific Company, Catalog J150, Item 83605.

² Heath Company, Model V-6 Vacuum Tube Voltmeter.

LETTERS TO THE EDITOR

The Concept of Radiation Measurements

REFFERRING to the interesting paper by D. J. Lovell,¹ I wish to point out a slight error in the description of the radiance of a surface as listed in Table I.

Following the notation of the author as given on page 461, the symbol for the radiance of a surface written without the subscript is understood to mean the radiance of a diffuse reflector. Assuming that the author intended the above in Table I, it would then appear that a more exact description of the radiance should be: the flux per unit solid angle emitted per unit area from an infinitesimal element of surface in the direction normal to the surface. This definition follows directly from Lovell's Equation (4). However, it is not a general definition since it applies only to diffuse surfaces. I am sure, however, that Lovell intended his description of the radiance to be a general description for any surface. I offer the following as a general, and thus preferable, definition of the radiance for any surface in any direction: the flux per unit solid angle emitted by an infinitesimal element of a surface in that direction per unit projected area of the element projected on a plane perpendicular to that direction. This essentially is the definition originally given by Lovell on page 460 of his paper, and agrees with the accepted definition² of luminance, the analogous photometric term.

I further wish to point out that only if one of the above definitions is taken is it possible to derive correctly the expression for the total flux emitted by a diffuse surface when given only the radiance of the surface.

W. L. STARR

Basic Sciences Research Department
U. S. Naval Civil Engineering Research and
Evaluation Laboratory, Port Hueneme, California

¹ Donald J. Lovell, Am. J. Phys. 21, 459 (1953).

² International Commission on Illumination: "Definitions," J. Opt. Soc. Am. 41, 734 (1951).

Basic Physics Courses in Junior Colleges
and Technical Institutes

HOW do introductory physics courses for students in junior colleges differ from the conventional introductory courses offered to engineering students and physics majors in colleges and universities? Do physics courses in technical institutes treat more specialized topics and use more advanced mathematical analysis than do the introductory physics courses in colleges and junior colleges? What textbooks have instructors chosen? What features do they find particularly helpful in a textbook? What improvements are suggested?

Answers to these and other questions relevant to physics teaching were sought in a survey of basic physics courses in junior colleges and technical institutes (including all those Institutes accredited by the Engineers' Council for Professional Development).

Replies show that instructors of physics in junior colleges are using conventional college physics textbooks, for the want of a book more relevant to their objectives in many cases. In 40 junior colleges, where one might expect similarity of purpose, 26 different textbooks are used. The technical institutes generally strive to offer physics courses comparable to those in universities and engineering colleges. These institutes appear to be well satisfied with standard engineering physics textbooks which employ calculus. Here the diversity of choice reflects the specialities of the Institutes (such as aeronautics, electronics, and textiles).

In the most decisive expression of preference, instructors desire a cloth-bound physics textbook of conventional format, which omits student experiments, but includes a review of trigonometry. Other preferences are less well defined. Equal numbers of instructors regard the use of equal conventional current as highly desirable, immaterial, or undesirable. A majority favor stressing British engineering units in mechanics, and metric units in mks form (rather than cgs) when used as a basis for commonly-encountered units in electricity.

A distinctive characteristic of physics courses in junior colleges seems to be a laudable attempt, or at least the wish, to restrict the number of topics to a number of basic principles really manageable in an introductory course. Most instructors are willing to omit meteorology, color specification, and health physics, and to minimize acoustics.

In critical evaluation of individual textbooks, strong emphasis was placed on the desirability of many numerical problems (with answers), graded in complexity in each group.

Half the courses in introductory physics use no 16-mm instructional films; few make extensive use of them. Only two instructors mentioned the AAPT-sponsored College Physics series.

A summary of results of this survey has been prepared at the request of some of the participants. Available copies will be supplied to others interested, upon request to the writer.

ROBERT L. WEBER

The Pennsylvania State University
State College, Pennsylvania

On Demonstrating Bernoulli's Principle

THIS note is not intended to provide anything substantially new as a demonstration for the experiment is well known and much used. Rather it is intended to bear out a philosophy of demonstration experiments.

Whenever I can, I make a demonstration experiment a large-scale affair. It is thereby seen easily everywhere in the room and possesses besides a measure of drama—a quite necessary ingredient. For the case at hand I use a glass tube (cylinder) some 6 or 8 feet long, and an inch

or so in bore, its lower end in a beaker of water colored with eosin. With a sharp blast of air across the upper end one can get an elevation of water in the tube some 2 or 3 feet.

Now on a recent occasion I remarked to my colleague that the effect would be more pronounced with a tube of smaller bore—say one-half inch across, whereupon he frowned somewhat and reminded me that the pressure was governed by the *elevation* of the water, not its cross section! This I happily admitted but I showed him promptly that with a smaller tube the elevation was indeed greater. We both agreed on the apparent dilemma and

its obvious answer. But now I showed it to the class and it incited sharp debate.

What I am saying here is this: a demonstration experiment has substantially more value if dilemmas are introduced. The experiment should, of course, demonstrate a principle but if it does *only* this little thinking is incited. After all, we expected the results and there is little to think about. If, however, the situation permits what I call an "alien twist" the students go away debating and discoursing—and thereby learning more physics.

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AMERICAN JOURNAL OF PHYSICS

VOLUME 22, NUMBER 3

MARCH, 1954

RECENT MEETINGS

Southern California Section

Sixty-five members and as many guests attended the annual Fall meeting of the Southern California Section of AAPT on October 10, 1953. The invited paper on **Modern Electronics** by Dr. SIMON RAMO was interesting, provocative, and prophetic. The conjecture on "synthetic intelligence" proposes as great a revolution in human affairs as the Industrial Revolution. Following Dr. Ramo's talk a panel discussion on *Improving High School Mathematics* was engaged in, with Dr. Willard Geer presiding. The principal problem was a consideration of the usual formal course in solid geometry with the recommendation from the AAPT that this semester's work be revised. The problem is to be pursued with the mathematics people here in Southern California. After lunch Dr. David Bender of Whittier College, President of the Section, introduced the presentation of the following papers.

1. Photoelastic strain patterns without polarizers. ALBERT V. BAEZ, *University of Redlands*.—Light from the northern sky and a horizontal reflector such as glass or plastic produce the same effect as a polarizer and analyzer. Doubly refracting materials such as cellophane and plastics are viewed by reflection and exhibit the characteristic colors associated with the familiar polarization experiments. One advantage of using the sky is that very large objects may be viewed. Color photographs are exhibited to illustrate several interesting effects.

2. Measurement of the change of phase due to scattering. HERBERT SMITH AND ROBERT WITT, *University of Redlands*.—The mathematical expression for the diffraction pattern of a point object illuminated by a monochromatic point source is derived, allowing the possibility of a phase change due to scattering at the object. Theoretical predictions as to the magnitude of this shift are described, and experiments designed to detect and measure it are explained. Results are given which confirm the theory in part.

3. Rumors in a physics class. WALTER O'CONNELL, *Mt. San Antonio College*.—The study of physics can pro-

foundly alter all of our attitudes toward the world, but there is a danger here of starting unfounded rumors. Physics is but one very limited way of examining the universe.

4. Gas tube gimmicks. FRANK PETRY, *West Los Angeles*.—Cold cathode gas tubes, owing to their relatively small size, simplicity of construction, and versatility of operation, can be utilized in a variety of instructive applications in the laboratory or classroom.

5. Are we neglecting optics and wave motion in general physics? ARTHUR W. NYE, *Los Angeles College of Optometry*.—Los Angeles is an important optical center. The local optical societies have representation from some 40 concerns designing and manufacturing optical parts. Students need training in these topics.

6. What physics has meant to me. W. P. BOYNTON, *Whittier College*.—Physics is fundamentally an intellectual discipline. While the physicist is not a professional philosopher he cannot escape some speculation as to the philosophical implications of his knowledge.

7. Fluid circulations and the motions of particles in the Kundt tube. HERMAN MEDWIN, *University of California and Los Angeles City College*.—Recent work in acoustics enables us to discuss the fluid circulations in the Kundt's tube and to predict the dust patterns which result from acoustic radiation pressure. The striations are accounted for.

8. Transformations from wave equations and vice versa. JACK C. MILLER, *Pomona College*.

At the business meeting the officers considered speakers for the Spring Meeting, which will be held at Occidental College.

JULIUS SUMNER MILLER
Secretary-Treasurer

Central Pennsylvania Section

The Central Pennsylvania Section of the American Association of Physics Teachers met at Bucknell University on October 23-24, 1953. This was the organization

meeting for the Section. Twenty-three colleges and three high schools were represented.

Ten contributed papers and seven invited papers comprised the program. There were also exhibits of scientific equipment and books.

Invited Papers

Topic: What can the small college do in the way of a modest research program in physics?

1. RAYMOND J. SEEGER, Acting Assistant Director, *National Science Foundation, Washington, D. C.*

2. DOROTHY W. WEEKS, *Wilson College, Chambersburg, Pennsylvania.*—Since the summer of 1935, the Department of Physics at Wilson College has attempted to carry on in a small way research in spectroscopy. After one of the faculty had received a summer of training in M.I.T.'s spectroscopic laboratory, Professor George R. Harrison suggested a study of the iron spectrum. There was a need for more iron standard wavelengths. In the fall of 1935 Professor Harrison undertook a W.P.A. project in spectroscopy which needed these additional standards. This W.P.A. project lasted until the U. S. entered World War II.

Before the project ended, the Bitter magnet was in operation, and Zeeman patterns of many elements were recorded on photographic plates taken at M.I.T. Persons better trained than most of the W.P.A. workers were needed to solve these patterns. It was difficult for students to find jobs during the summers of the depression years. Professor Harrison agreed to the suggestion that women students in physics be recruited to work for six weeks in the summer at no pay but with the privilege of attending his lectures in the summer course in *Practical Spectroscopy*. Therefore for the summers of 1939, 1940, 1941 such groups were recruited. After the war this program was revived for the summers of 1946, 1947, and 1948. A total of 27 girls from nine different women's colleges had this experience. Twelve of these 27 were students from Wilson College.

In 1936 a course in spectroscopy was introduced at Wilson College. In this course each student devotes about half of her laboratory time to a project. For example, a student of biology studied the chlorophyll content of different plants by studying their absorption spectrum; a premedical student studied the absorption spectrum of blood treated with different poisons; a chemistry student studied the impurities in coal obtained from four different mines in Pennsylvania. Students have found spectroscopically the elements in citrus fruits, rare earths in hickory leaves; the broadening of spectral lines when the source of light is in a strong magnetic field; and the fluorescent effects of waxes.

Students who were receiving this training were able to contribute to the program at M.I.T. and also to assist in computations carried on at Wilson College in the study of Zeeman patterns. Grants from foundations made available research assistants. The summer experience at M.I.T., the presence of research assistants in the department, the additional equipment made possible through grants for research, and the course in spectroscopy stimulated an interest in physics. In a study made of the enrollment in seven colleges for women including Wilson for 11 years

from 1939–1952 except for the years from 1945–1947, it was reported that Wilson graduated 20 majors in physics. Two of the seven associated colleges for women graduated 21 each, one 34 and another 39. All these colleges had larger enrollments than Wilson, in general more than double. The college graduating 39 majors in physics had an enrollment of more than four times that of Wilson College. In 1940–1941 only 27 of the 16 000 students enrolled in 23 colleges for women were senior majors in physics. The proportion of majors at Wilson College is more than double this average for the 23 colleges.

It is questionable that the interest in physics at Wilson College would be what it is, had the department not undertaken a small research project. For with its limited space for the physics department it would not have been possible to have set aside very much of any space for an experimental research project. Moreover, the fact that the research project was associated with M.I.T., and that for six years the able juniors during the summer following their junior year had the opportunity to spend six weeks at M.I.T., brought invaluable intangibles to the students and to the department. For example, students began to appreciate the vast amount of research work behind a simple statement in a textbook.

3. CHARLES H. SCHAUER, Director, Division of Grants, *Research Corporation, New York, New York.*

4. W. D. WALKER, JR., *University of Rochester, Rochester, New York.*

5. ROBERT F. McCUNE, *Bucknell University, Lewisburg, Pennsylvania.*

In addition to the above invited papers there were the following:

6. Application of microwave frequencies to communications. NELSON B. THARP, Section Manager, *Microwave Engineering Department, Baltimore Works, Westinghouse Electric Corporation.*

7. What is ultrasonics? HAROLD K. SCHILLING, *The Pennsylvania State University.*

Contributed Papers

8. Electrical units and the undergraduate physics course. THOMAS P. MERRITT, *Albright College.*—A study of the electricity and magnetism sections of several of our new and recently revised general physics textbooks reveals a certain amount of inconsistency among them in connection with the definitions of electrical units. Various examples of these inconsistencies are given. Finally a consistent set of electrical units involving the fundamental units of mass, length, and time is worked out in the esu, emu and mks systems, and the similarities and differences are pointed out. In this analysis the ampere is defined so as to take on the units of newton¹, which gives the permeability no units, but the permittivity takes on the units of sec² m⁻². It is shown that **B** and **H** have the same units whereas the units of **D** and **E** are different.

9. Teaching the principles of correct technical graphs. PEARL I. YOUNG, *Pottsville Pennsylvania State Center.*—

On account of the great emphasis placed on charts and graphs in modern research and industry, they should be correctly taught and used during all four years of a student's undergraduate program. By graduation he should be able to construct a correct circle, bar, area, and volume charts; draw alinement charts, nomographs, and trilinear curves. Beginning early in the general undergraduate physics course graphs should be constantly used in lectures and discussions. In laboratory reports the student should be required to plot his data in approved form, obtain the physical slope, and interpret the form and intercepts.

10. Measurement of the speed of light in the advanced undergraduate laboratory. C. HARVEY PALMER JR., AND GEORGE S. SPRATT, *Bucknell University*.—The purpose of this work was to assemble a laboratory experiment on the speed of light suitable for an undergraduate optics course and involving a minimum of expense and equipment. Apparatus using essentially the method of W. C. Anderson was constructed. Light from a small projection lamp is modulated by a Kerr cell operated with an adjustable dc bias of about 2.5 kv together with a 20-Mc high voltage. The interferometer has a 5-meter fixed path and a 3-meter variable path along an optical bench. The signal is at its minimum when the movable mirror is approximately half way along the optical bench, the minimum point being determined to within 1 cm. The output voltage of a 931A photomultiplier is heterodyned to 6 Mc and fed into a short-wave receiver. The optical beam is mechanically interrupted at 900 cps to give an af modulation which appears on an oscilloscope and is measured with a vacuum-tube voltmeter. Results as measured have been within 1 percent of the accepted value and would probably be within 0.1 percent if simple precautions in shielding were taken.

11. Some experiments for advanced laboratory. JOSEPH P. HARPER, *University of Scranton*.—Some of the experiments included in the advanced physics laboratory course are described. Most of the experiments can be classified as electronic, optical, thermal, or nuclear in nature. The emphasis is on the experiments for which the equipment was designed and built by faculty or senior physics students. This equipment includes a comprehensive nuclear counter. The experiments described include: measurement of the index of refraction of a gas; measurement of the emissivity of metal surfaces; absorption of gamma rays; range and energy of beta rays.

12. A visual aid for the presentation of wave forms to first-year students. W. NORWOOD LOWRY, *Bucknell University*.—A wave template set (Cenco Cat. No. 84782, Cat. J-150) was used to draw permanent diagrams of wave forms on ordinary large white window shades mounted on spring rollers, which are hung in the front of the lecture room. Colored inks are used in a systematic manner to identify identical wave forms. The equations for each wave and their combinations are placed at appropriate positions. A small scale in the shape of a try-square, marked off in contrasting-color "inch" bands and mounted on a long

stick, permits the instructor to show the students the amplitude and wavelength of each wave form. By this means it is possible to demonstrate quantitatively the resultant wave form of two simple sine curves, for a number of different combinations. Two window shades, 8×8 ft and $6 \times 7\frac{1}{2}$ ft, were used.

13. A senior honors thesis program. F. W. VAN NAME, JR., *Franklin and Marshall College*.—The senior honors thesis program of Franklin and Marshall College is described. The work done in three recent theses is reported, and the program is evaluated. The extension of the program to include other seniors on a voluntary basis is suggested.

14. A simple transistor experiment. N. P. BOSTED, *Bucknell University*.—An experiment for the undergraduate electronics laboratory in which measurements of transistor parameters by static and dynamic means are made. All equipment employed is such as would be found in the average laboratory. Suggestions are made for further work.

15. Neutron cross sections at long wavelengths. VERNON W. MYERS, *Pennsylvania State College*.—The slow neutron time-of-flight apparatus at the Brookhaven National Laboratory is described. The total neutron cross sections of heavy water and boron-free glass have been measured in the wavelength interval 4–15 Å. These cross sections exhibit the coherent scattering expected in liquids. Inelastic scattering predominates at the longer wavelengths.

16. Physicists-in-training during the next ten years. MARSH W. WHITE, *Pennsylvania State University*.—There has been a sharp drop in the numbers of physicists-in-training since 1950. This decline is most pronounced for students at the bachelor's level. Enrollments of graduate students in physics have decreased only slightly, but this decline will continue until about 1958. During the academic year 1952–1953 the production of Ph.D.'s in physics reached an all-time high of 525. This number will slowly decrease until about 1962, following which there should be a considerable upturn. Data on the enrollment of and degrees awarded to physics majors for the next ten years have been estimated on the basis of the present secondary school population, with the assumptions made that the trends of the last few years constitute a reasonable basis for predicting future enrollments. Such highly tentative predictions indicate a production of B.S. physics majors to continue for several years at about the present annual rate of 2500. Similarly the number of master's degrees in physics will continue at about 800 for the next five years. The number of physics Ph.D.'s will probably decrease from the present 500 to about 400 per year. After this five-year decline it seems likely that there will be small, but steady, increases in physics majors at all levels for the following five years.

17. Laboratory work in atomic physics. HARRY L. RAUB, *Muhlenberg College*.—Atomic physics is offered as a second-year physics course designed to interest students majoring

in other fields as well as physics majors. The laboratory is made an integral part of the course with the experiments being correlated with the recitation so that they can be illustrative of classroom discussions. The level of the experiments is set so as to hold the student's interest and improve his technical ability. A majority of the apparatus is designed in the department and constructed by full-time technical help with student assistants, in an attempt to make the best use of a store of equipment acquired in recent years. The student is encouraged to work with a minimum of faculty assistance, with references to original source material being offered as guide for his thinking.

DOROTHY W. WEEKS
Secretary

Oregon Section

The sixty-third meeting of the Oregon Section was held at Lewis and Clark College, Portland, Oregon, on October 31, 1953. The activities of the day included a business meeting, a group luncheon, and a tour of the laboratories. The following program was presented.

1. Rate of sedimentation in the Skagit Bay region. L. A. SANDERMAN AND H. R. BLANK, JR. *University of Washington.*—It has been found by most investigators that the distribution of radium with depth below the surface of the ocean bottom shows an increase of radium concentration with depth followed by a gradual decrease. This distribution has been explained on the basis that the uranium, ionium, and radium present in the sediment consist of these elements originally present together with that extracted from the sea water. It has been found that there is more ionium in the sediment than is necessary to maintain radioactive equilibrium with the radium and less uranium than is necessary to maintain equilibrium with the ionium. If it is assumed that a constant ratio of the above elements has been maintained, it is possible to calculate the rate of deposition of the ocean sediment from an analysis of the variation of the radium concentration with depth. This method of measuring the rate of deposition of sediment has been applied to two inshore cores taken from the Skagit Bay region of Washington. Both cores yield an average rate of deposition of 20 years per centimeter of wet sediment or 50 centimeters per thousand years. Until further measurements are made, these results are considered preliminary.

2. New radioactivities of gallium.* BERND CRASEMANN, *University of Oregon.*—Using a "trochoidal analyzer" for their detection, positrons were found to be associated with the decay of 15 minute Ga^{65} . This fact has also recently been noted by Dutch workers.¹ By means of a thick lens beta-ray spectrometer, the positrons were resolved into two groups of 2.52 ± 0.05 and 2.1 ± 0.1 Mev maximum energy and of 10 percent and 90 percent abundance, respectively. The more energetic group, with $\log f\tau = 6.0$, fits into the class of "one-forbidden" transitions, recently defined by Feenberg and Trigg.² This agrees with the shell

model. An isomeric level of half-life 8.0 minutes in this isotope has been identified through excitation curves and cross-bombardments. It presumably lies below the 15-minute level and is fed by a highly converted 52 kev gamma ray. The new isotope Ga^{64} has been produced in the Crocker Laboratory cyclotron by the reactions $Zn^{64}(p,n)Ga^{64}$, $Zn^{64}(d,2n)Ga^{64}$, and $Cu^{63}(a,3n)Ga^{64}$. It decays by positron emission with a half-life of 2.6 ± 0.1 minutes.

* Work done at the University of California, Berkeley, California.
¹ Aten, de Wijs, and Boelhouwer, *Physica* **18**, 1032 (1952).

² E. Feenberg and G. Trigg, *Rev. Modern Phys.* **22**, 399 (1950).

3. Cyclotron beam energy determination by a time-of-flight method. J. H. MANLEY, *University of Washington.*—A simple time-of-flight method has been employed as a means of measuring cyclotron beam energy. Through use of a movable and a fixed stilbene scintillation detector, a phase delay measured in terms of the cyclotron frequency is introduced by the flight-time of the beam particles. Results good to 2 percent in energy have been obtained. These have less precision than the direct errors of the method, largely because of cyclotron instabilities.

4. The electrical response of the human eye; a note on its variation with dark adaptation. MATHEW ALPERN AND JOHN J. FARIS, *Pacific University.*—A technique for measuring the electrical response of the living human eye to a flash of light is described in detail. The magnitude of this response increases with time in dark. Previous data¹ indicated that with moderate intensities of prior light adaptation and stimulus flash the principal component can be related only to rod activity. The present experiments (using intense levels of prior light adaptation and stimulus flash) show that when magnitude of response is plotted as a function of time in the dark, a clear break in the curve appears at seven minutes in the dark. This break in the curve resembles the rod-cone break in the subjective dark adaptation curve in that reducing the intensity of prior light adaptation eliminates it. Electrical light adaptation curves resemble the subjective light adaptation curve of the cones, also implying the principal component of the electrical response can reflect cone activity.

¹ E. P. Johnson, *J. Exptl. Psychol.* **39**, 5, 597-609 (1949).

5. Report on the Danforth seminar. J. DAY, *Oregon State College.*—A seminar on the "Teaching of Natural Sciences in Relation to Religious Concepts" was held at the Pennsylvania State College, July 6-17, 1953. Financial support was given by the Danforth Foundation; facilities were provided by the College. The seminar was led by Dr. W. G. Pollard, Executive Director of the Oak Ridge Institute of Nuclear Studies. The following summary statements seem justified as general conclusions of the group of 26 experienced science teachers in attendance: (1) Orientation in philosophy, theology, and early Hebrew-Christian history is as important for a science teacher as is orientation in science for teachers of philosophy, theology, and the humanities; (2) There is an acute problem of effective communication of ideas between specialists in science and

theology; (3) An historical and critical interpretation of the Hebrew-Christian drama resolves many so-called conflicts between science and religion; (4) Science is capable of reinforcing rather than tearing down Christian conviction; this can be and should be done without ignoring either established fact or areas of current disagreement.

6. Vector solution of network problems. D. D. LOOMIS, *Lewis and Clark College*.—This paper intends to give a simple vector solution to the T network with practical application of determination of component parts for any impedance ratio or phase advancement or retardation. This solution is especially adaptable to students in the junior or senior classes.

7. The effect of temperature on field emission.* W. W. DOLAN, W. P. DYKE, J. K. TROLAN, AND J. P. BARBOUR, *Linfield College*.—Both the current density and the distribution in energy of electrons emitted from metals are calculated for various combinations of temperature, applied surface electric field and work function. A wider range of those variables than previously achieved is made possible by use of numerical integration. The integrand is the usual function based on the free electron theory of metals and the wave-mechanical barrier transmission coefficient of Sommerfeld and Bethe which assumes a classical image force and a plane surface. Results, which are presented in graphical form, are consistent with the Fowler-Nordheim field emission equation for low temperatures, and with the Richardson thermionic emission formula at low fields. Predicted emission at temperatures up to 3000°K is compared with cold emission at fields between 10^7 and 10^8 v/cm. The calculated current densities are confirmed experimentally for a work function of 4.5 ev, at fields from 2.6 to 6.8×10^7 v/cm, and temperatures up to 1400°K at the lower fields and up to 2000°K at the higher fields.

* This work was supported by the U. S. Office of Naval Research

8. Resistivity striations in germanium crystals. PAUL R. CAMP, *Reed College*.—Germanium crystals for use in devices are usually prepared containing a concentration of some desired impurity of the order of 10^{-6} to 10^{-9} impurity atom of germanium, the specific value depending on the device to be made. Most methods of preparation result in a gradual change of impurity concentration along the length of the crystal as the result of ordinary segregation processes. In addition, there frequently occur sharp fluctuations in the impurity concentration superimposed on this slower change. A rather simple technique for detecting fluctuations of this kind will be described. It is more sensitive than resistivity scanning and does not involve the use of radioactive tracers. The origin, value, and elimination of these fluctuations will be discussed briefly.

KENNETH E. DAVIS
Secretary

Kentucky Section

The annual fall meeting of the Kentucky Section of the American Association of Physics Teachers was held

on October 31, 1953 in Pence Hall of the University of Kentucky. The meeting, held in conjunction with the Kentucky Academy of Science, was attended by 39 members and guests. DR. CARL ADAMS, *University of Louisville*, presided. Following the program of contributed papers an informal luncheon was held in the Student Union Cafeteria.

The contributed papers were as follows:

1. Nineteenth century experiments in magnetism. J. R. LAMARSH, *University of Kentucky*.—A discussion of several experiments in magnetism carried out in the last century, certain of which have not been repeated recently.

2. Keep your eye on the architect. D. M. BENNETT, *University of Louisville*.—Some of the headaches involved in the construction of a new science building.

3. The combined Physics-Radiology program at the University of Louisville. DONALD W. HENDRICKS AND D. M. BENNETT, *University of Louisville*.—The purpose of this paper was to explain the new physics-radiology program at the University of Louisville, set up by the Physics Department in conjunction with the Radiology Department. It was pointed out that the ultimate aim of the program was to offer both a B.S. degree and an M.S. degree in Biophysics. Mention was also made of similar programs at other universities.

4. Some comments on physics according to Aristotle. WALDEMAR NOLL, *Berea College*.

5. A generalization of a theorem by Pappus. E. E. MAYO, *University of Louisville*.—This paper pointed out that the integral referred to in the theorem of Pappus was essentially

$$I_{pqr} = \int_B x^p y^q z^r dm,$$

where $p=q=r=0$ and \int_B means integration is extended over the entire body B . We chose to discuss the above integral with p , q , and r all integers greater than or equal to zero, and define B as the body generated by revolving a surface S with parametric representation in $(\alpha\beta)$ about the z axis, which is external to S . Then the above integral reduces to a finite sum of integrals similar to I_{pqr} , but involving only S .

As a sample case, the hollow cylinder formed by revolving a rectangle about the z axis was used to illustrate the use of the formula in computing the volume, mass, and moment of inertia about the z axis.

6. Matrix algebra for undergraduates. PAUL C. OVERSTREET, *Morehead State College*.—There are certain problems in analytical geometry, calculus, and the various branches of physics offered to college students whose solution is more elegant, compact, and comprehensive when the simpler uses of matrix algebra are employed.

Such problems are: finding the equations of curves which meet certain geometric conditions, finding the precession of a spinning top, solving network problems in current electricity, translating velocities and accelerations from rectangular axes to polar axes.

It is not necessary to teach the student matrix theory. All he needs is to be shown how to express his problem in matrix form, how to find matrix products and how to develop a reciprocal matrix. Some of my students have

developed quite an interest in the use of matrices in solving their problems.

LEWIS W. COCHRAN
Secretary

Michigan Teachers of College Physics

Sixty-four teachers registered for the Autumn meeting of the Michigan Teachers of College Physics at Michigan State College in East Lansing on Saturday, November 7, 1953. Morning and afternoon sessions were held for presentation of research reports and papers on elementary teaching. An exhibit of demonstration equipment and techniques developed by Professor Robert Pohl of Göttingen University highlighted the meeting. A luncheon was held at the Michigan State Union and an informal tea and social hour concluded the day's activities.

DEAN THOMAS H. OSGOOD, *Michigan State College*, served as chairman for the morning session; PROFESSOR E. A. HIEDEMANN, *Michigan State College*, was chairman of the afternoon program. Ten invited and contributed papers were presented as follows:

1. Weighing filaments by vibrational techniques. D. J. MONTGOMERY, *Michigan State College*.
2. An optical method for demonstrating the refractive index gradient in a sound wave. PAUL LOEBER, *Michigan State College*.
3. Pure quadrupole spectra of molecular crystals. H. C. ALLEN, JR., *Michigan State College*.
4. Recent work in liquid crystals. R. D. SPENCE, *Michigan State College*.
5. A small dc motor experiment for the elementary laboratory. GEORGE BRADLEY, *Western Michigan College*.
6. The conical pendulum—a demonstration and proof. ROSS GILLETT, *Jackson Junior College*.
7. Experiences in teaching home economics physics. E. F. CARR, *Michigan State College*.
8. Electrification of fibers. ROBERT CUNNINGHAM, *Michigan State College*.
9. Objects and images. C. D. HAUSE and WALTER GESSERT, *Michigan State College*.
10. Streamlines and other things. ALFRED LEITNER, *Michigan State College*.

ANNOUNCEMENTS AND NEWS

Book Reviews

Waves and Tides. R. C. H. RUSSELL and D. H. MACMILLAN. Pp. 348, Figs. 117, 14×22 cm. Philosophical Library, New York, 1953. Price \$6.00.

This work consists of two sections written separately by two authors and bound into one cover. The similarity between the two subjects, waves and tides, makes this arrangement desirable. Many concepts are developed by Russell in his section on waves upon which the understanding of the ocean tide is dependent, and so the waves section precedes that on tides in logical fashion.

This book, although very comprehensive, is not intended to be either a textbook or a treatise. It includes many aspects that workers in the field of waves and tides may never have delved into. Consequently, each chapter is fairly brief and to the point. The book is well referenced and will serve very well as a starting point for those who are interested in the theory, practice, or history of these subjects.

The book is written for the well-rounded layman. Accordingly, practical considerations are covered with more thoroughness than is theory. Only the principal aspects of classical wave theory are presented, and their uses and limitations discussed. The intuitive approach rather than mathematical symbolism is used, and it is developed to the extent where the reader can gain some insight into quite complicated mechanisms. The reader with a general background in physics will not be at all impatient with the nonmathematical nature of the book, but will more than likely acquire many new ideas on the subject.

This book brings out a striking discrepancy between the relative efforts that have been expended in recent years on studies of waves and tides. Except for the work of Doodson and his associates, there has been little in the way of progress in the subject of tides for 50 years. The study of waves, however, is being actively pursued in England, United States, and France, and the book is as up-to-date as could be expected under the circumstances. Some of the very recent work following a statistical "noise" approach necessarily has been left out.

The chapters "Movement of Material by the Sea" and "The Effect of Wave Action on Structures" contain the results of several investigators presented very concisely. The readers should bear in mind, however, that the cited findings of some of the investigations are still subject to controversy.

The section on tides is remarkably understandable considering the difficulty of describing such a complex subject. Novelty has been sacrificed for straightforwardness of explanation, and the readers who have had previous contact with tides will certainly recognize many of the diagrams. The peculiar tidal characteristics of many regions of the world are described and explained. Although each region has its own particular response to the tidal forces it is possible to form many generalizations relating the topography and location to the observed tidal effects. The author has pointed these out utilizing the results of theoretical investigations dealing with idealized basins and channels.

The inclusion of many diverse topics has, in the reviewer's opinion, added substantially to the value of the

book. The title, perhaps, might have been *Ocean Surface Waves and Ocean Tides* for the authors confined themselves exclusively to the discussion of ocean phenomena. Nothing was said about other wave phenomena nor atmospheric tides, body tides, etc. The plates are very good as well as informative, and the excellent combination index and glossary at the end of the book should prove most helpful to the readers.

GORDON W. GROVES
University of California

Principles of Color Photography. RALPH M. EVANS, W. T. HANSON, JR., AND W. LYLE BREWER. Pp. 698+xii, 15×23 cm. John Wiley and Sons, Inc., New York, 1953. Price \$10.00.

This is a book for those who are already familiar with the general subject as presented in Ralph M. Evans' admirable work, *An Introduction to Color*. In *Principles of Color Photography*, the authors correlate and organize the vast amount of research done in this and related fields. The writing has covered the past fifteen years, a period that has seen the subject develop from a largely empirical aggregate into something whose basic principles are now much more clearly understood.

This book represents a very carefully written and thoughtful summary of what is known and also includes a large amount of previously unpublished work. An extensive bibliography is included. The following partial list of chapter headings is representative of the scope and general character of the treatment given: Visual Processes and Color Photography, Photographic Formation of the Color Image, Color Sensitometry, Optical Characteristics of Colorants in Combination.

The authors have resorted to a mathematical treatment whenever they have considered this desirable in the interest of scientific rigor but have sought to develop the argument in these cases so that the general trends and conclusions can be comprehended by those who lack the background or inclination to submit to the discipline of special mathematical devices such as matrix algebra.

While color processes are chemical as far as the formation of the dyes is concerned, the principal emphasis in this book is placed upon the optical effects produced after they are formed and upon the psychophysical and psychological aspects involved in the interpretation by the observer of what he sees.

In fact, it is possible to inquire amiably whether we shall reach the end of the journey in color photography until we are in position to set forth the laws of optical illusion. Examples occur occasionally both in color photography and in painting that cause us to think we see what is not there. If the photographer or artist provides us with a good set of clues we are able to contribute a good deal out of our own experience. The same sweep of a brush filled with brown paint may be the bow of a boat resting on the surface of a body of water or a rock below it depending upon where the water line is placed.

A short time ago I saw projected a small Kodachrome slide that had been taken from the deck of an ocean liner

just as it was approaching the dock at a foreign port. The view shows fully two hundred people waiting there and one gets a curious sense that they are individuals and not merely an indistinct mass of humanity in spite of the numbers. Somehow it is possible to single out individuals and get a distinct idea of what they are doing and wearing. A tiny dot of red of indeterminate shape becomes to the observer a red jacket. One of white is a waving handkerchief and so on. Just how much of this is due to an unusually happy combination of light and color contrast and how much the observer puts into it out of his own experience, it would be difficult to say. At least, it seems probable that our minds accept the symbols provided in such cases and add considerable that is not there.

In other optical illusions, the elements provided may cause us to obtain a perfectly preposterous impression that is completely outside the world of reality. In a recent lecture on 3-D projection before the Optical Society of America, Dr. Land projected an image of a house fly so that it appeared to be about three feet long and to hang suspended above the center of the auditorium. There is ground for argument that the world of sober scientific fact joins forces with the world of fantasy in color photography.

In this book it is very heartening to discover the wide range of subjects covered and to realize the patient enterprise represented by all these investigations. Sometimes it seems as if physicists forget that our field is natural philosophy. We are inclined to concentrate our efforts in limited areas, rapidly to exploit them to exhaustion, and then move on to do the same thing somewhere else. Sometimes these limited areas are thrust into such prominence as to seem to imply that they are the whole of physics. This book will offer a very rewarding antidote to combat this tendency to all who seek exact and comprehensive knowledge of the vast domains in physics that contribute to color photography.

KATHERINE CHAMBERLAIN
Wayne University

Physics, Principles and Applications. Second Edition.

HENRY MARGENAU, WILLIAM W. WATSON, AND C. G. MONTGOMERY. Pp. 814+xii, Figs. 631, 16×23.5 cm. McGraw-Hill Book Co., Inc., New York, 1953. Price \$7.50.

This is a rather extensively revised edition of a textbook designed to be used in a year's course at the sophomore level of college. The chief revision has been in the sections on electricity and magnetism where the material has been rewritten with somewhat less rigor than in the first edition. The authors retain their segregation of the more difficult portions from the rest of the text by using smaller type. Approximately twenty percent of the text is so treated and the authors assume that the individual instructor will tailor the material to fit the needs of his class.

The calculus is used sparingly at first, reaches a rather high crescendo with the solution of the differential equations for damped simple harmonic motion and for forced oscillations at a point one-fourth of the way through then

declines to a very reasonable application throughout the remainder of the book.

The exposition is clear and in general indicates a breadth and depth of understanding that many books of this level do not equal. The volume is well illustrated and indexed and answers are given in the back.

Looking at this book with a critical eye, much can be found to recommend it for a sophomore course. Certain errors have crept in, as I suppose is inevitably the case with all such books. It may be worth while calling attention to the following.

On p. 466 is given the charge that flows through a ballistic galvanometer, when a current Δi_1 is changed in one coil of a mutual inductance. The answer is correct but in setting up the expression for the current in the secondary circuit the self-inductance should be included.

On p. 477 a quite common mistake is made in the definition of diamagnetic materials. Here it is stated that, "When a bar of *diamagnetic* material is placed in a magnetic field, a torque is exerted that tends to set the bar at *right angles* to the field, since, it can be shown, in this position the energy is a minimum. From this phenomenon the name is derived, the prefix *dia* meaning across." Also, "A *paramagnetic* rod tends to set itself *parallel* to the lines of magnetic field." (Italics mine.) Actually in a uniform magnetic field (here implied), both experiment and theory show that each kind of material, if elongated, will tend to set itself parallel to the magnetic field. It is rather surprising that this mistake has been propagated from textbook to textbook. The definition of diamagnetic and paramagnetic materials can probably best be stated in terms of the forces exerted on the body in a diverging magnetic field.

On p. 768 it is stated that the transformation of hydrogen into helium in the sun "can continue for at least 10^7 years." This is probably very much too conservative. The accepted figure for the life of the sun is more nearly 10^{11} years. Present evidence is that the sun has used up only a few percent of its hydrogen in the past 4×10^9 years.

When the authors state in the preface that they have included a "plenum of material" they are not making a false claim. The propagation of second sound in liquid helium and quantum electrodynamics are among the few recent developments in physics that are not mentioned. The wise teacher will have to use considerable judgment in choosing the material for which his students are to be held. The book is obviously written for the student who needs a speaking acquaintance with many of the more recent developments in physics and at the same time, some appreciation for the analytical methods that have been fundamental to the progress of physics.

H. V. NEHER
California Institute of Technology

Motion Pictures for Classroom Use

The Visual Aids Committee of the American Association of Physics Teachers is cooperating with the McGraw-Hill Book Company in the production of the College Physics film series. Each film is about eight minutes long

and is designed to accomplish one or more of the following objectives: (1) To fill in the gap in a demonstration lecture created by the absence of suitable demonstration equipment. (2) To explain the behavior of equipment which, if actually demonstrated, operates too rapidly or in too complicated a manner. (3) To make the textbook "come to life" by providing animated derivations of particularly important equations.

The films contain no extraneous material, music, or irrelevant photography. They are executed entirely in animation. Each film deals with one topic as a complete unit. The narration is brief and to the point.

At the present time six films in the series are available.

Uniform Circular Motion. Price, \$37.50. Uniform circular motion is defined and demonstrated. Changes in the velocity vector are shown and are used to explain why this motion is accelerated motion. The expression for the magnitude of the centripetal acceleration is derived. The meaning of centripetal force is explained in general, and illustrated in connection with a body whirling on a string, a car on a loop-the-loop, gravitational attraction, and the motion of an electrically charged particle in a magnetic field.

Gasoline Engine (Otto Cycle). Price, \$37.50. The six processes that are repeated in the gasoline engine cycle are represented graphically: Intake Stroke, Compression Stroke, Combustion, Power Stroke, Valve Exhaust, Exhaust Stroke. Next, an ideal situation is demonstrated with such disturbing factors as friction, acceleration, turbulence, and heat conduction eliminated. The resulting cycle, known as the Otto cycle, is traced on a pressure-volume diagram. The concept of thermal efficiency is discussed. It is pointed out that even in ideal cases and with all efficiency-lowering factors neglected, efficiency cannot be one hundred percent.

Diesel Engine (Ideal Diesel Cycle). Price, \$37.50. The six processes that are repeated in the Diesel engine cycle are explained with the aid of diagrams showing the motion of a piston in a cylinder. Ideal conditions are assumed and the resulting ideal Diesel cycle is plotted on a pressure-volume diagram while the actual processes are taking place. Factors affecting efficiency are discussed.

Carnot Cycle (Kelvin Temperature Scale). Price, \$37.50. The four ideal processes constituting the Carnot cycle are explained by the motion of a piston in a cylinder along with a moving point on a pressure-volume diagram. The heats absorbed and rejected, respectively, during the cycle are displayed graphically and used to define the Kelvin temperature scale. The efficiency of a Carnot engine is discussed in terms of the Kelvin temperature of the source and of the reservoir.

Measurement of the Speed of Light. Price, \$37.50. The toothed-wheel method of Fizeau is described in detail, with the aid of a moving light beam. A modification of this method, due to Michelson, is explained by reflecting

the light beam from a rotating eight-sided mirror. The motion is slow enough so that the path of light may be understood at all times.

Doppler Effect. Price, \$37.50. The factors determining the pitch of a sound are first explained. Waves are shown advancing toward a listener and the following three cases are treated carefully: (1) Source and listener stationary. (2) Source in motion, listener stationary. The accompanying change of wavelength and consequent change of apparent frequency are described in slow motion. (3) Source stationary, listener in motion. It is shown that no change of wavelength occurs, but that there is a change of apparent frequency.

The following additional titles are in preparation: Simple Harmonic Motion. Progressive Waves (Transverse and Longitudinal). Stationary Transverse Waves. Stationary Longitudinal Waves, Part I. Stationary Longitudinal Waves, Part II.

When two films are purchased at the same time, the charge is \$70.00 for the two. The prices quoted apply only in the U. S. and Territories, and are subject to change without notice. They may be ordered from McGraw-Hill Book Company, Inc., Text-Film Department, 330 W. 42 Street, New York 36, New York.

Practical Aids for Teachers of Physics

Improvisations

Professor Louis R. Weber, Colorado A. & M. College, Fort Collins, Colorado, spent the year 1952-1953 as Fulbright Lecturer in the College of Arts and Sciences, Baghdad, Iraq. He found himself much less liberally supplied with demonstration equipment and convenient storage rooms than in the U. S. He therefore improvised many lecture demonstrations in which student participation was essential. Here are some of his suggestions:

13. *Expansion:* Have six or eight students stand in a group at the front of the lecture room. Now if the students are asked to extend their arms periodically, they will unconsciously move away from each other.

14. *Virtual Image:* Place a student near the blackboard facing the class. After he is blindfolded, throw an eraser at the board so that it rebounds striking the back of the student. Ask him concerning the source of the eraser. (I was once accused of throwing erasers at students when it suddenly dawned upon me that the above demonstration was made the year previously.)

15. *Emf of a Cell:* The erasers are gathered into two piles on the floor in front of the lecture table. Three students are assigned to each pile. One student lifts the erasers to the lecture table; another student restrains him; while a third student places the erasers back on the floor. If this action takes place for about five seconds, there are invariably some erasers on the lecture table; say four in one pile and one in the second pile. The difference in the number of erasers corresponds to the electrical charges when two electrodes such as copper and zinc, respectively,

are placed in their respective salt solutions. The three students in each pile correspond, respectively, to the metals going into solution, the osmotic pressure of the solution, and the electrostatic layer in the metal.

16. *Antenna, Fundamental Frequency:* Have a student walk back and forth at the end of the lecture table while another student walks similarly along the length of the table. The difference in the number of round trips in a given time is quite apparent.

The Science Teacher 20, 114 (1952) carries an article by Samuel W. Bloom, Monroe High School Science Department, Rochester, New York, entitled, "Educationally, What does Industry Need?" Read it and discover that the decision as to how you will teach and what you will teach cannot be left until a student's senior college year.

Report of Activities of the Committee on Engineering Education

Your Committee on Engineering Education met in New York on October 2, 1953, with two committees of the ASEE, namely, the Executive Committee of the Physics Division and the Physics Subcommittee of the Committee on the Evaluation of Engineering Education. At this meeting it was decided by your committee that its immediate efforts should be devoted to the task of offering to physics teachers an opportunity to discuss at length the role of physics in engineering education.

The rapid change in the nature of the science of physics and in the perspectives of the engineer has stimulated extensive examination and criticism of the teaching of physics to engineers. It seems to your committee that physics teachers should hear the various views to be found among engineers and should be heard concerning proposed changes while new policies are in the process of formation.

Two steps have been taken toward this end. First, the chairman of your committee has joined with the chairman of the Physics Division of the ASEE in sending a letter to the heads of Physics Departments in the engineering colleges asking them and their staffs to consider the role of physics in engineering education and to report back their conclusions and recommendations.

Secondly, your committee arranged a session for the Annual Meeting of the AAPT on "The Responsibility of the Physics Teacher in Engineering Education." The speakers for this session are Dean L. E. Grinter, President of the ASEE, Dr. Homer L. Dodge of the Cabot Fund at Norwich University, Professor Nathaniel H. Frank of M.I.T., and Professor Lloyd P. Smith of Cornell University.

Plans for the future will be largely determined by our experience at this session of the Annual Meeting. One possibility, however, is that responses from heads of Physics Departments to the letter referred to above will indicate topics and speakers for subsequent sessions.

JOSEPH H. KENNAN
Chairman

To be published in April

INTRODUCTION TO COLLEGE PHYSICS

By Rogers D. Rusk
Mount Holyoke College

- Designed for college courses in general physics which do not presuppose a background in mathematics beyond high school algebra and geometry; the limited amount of trigonometry required is introduced in the text as needed.
- Includes all topics commonly studied in general physics courses; substantial attention is given to modern physics and electronics without sacrificing an adequate treatment of classical physics.
- Provides a careful blending of simplicity and clarity of statement with rigor of treatment—the product of twenty years of teaching experience.
- Furnishes many illustrative examples of the application of the basic principles of physics.
- Begins with English and cgs units; utilizes mks units where more desirable.
- Supplies numerous graded problems with a sufficient number simple enough so that the student acquires some facility before solving those of greater complexity.
- Contains a brief review of elementary mathematics in an appendix.

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